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On the Application of a Model of Mortality

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Abstract

A two parameter model of mortality is presented which is a simplification of an earlier three parameter model formulated by Mitra (1983). It expresses $\ell n(-\ell(x))$ as a linear function of $\ell n(x)$ and $\ell n(\alpha - x)$ where x is age, α is the upper limit of life and $\ell(x)$ is the probability of surviving from birth to age x. This is accomplished by constraining the model to reproduce the given value of infant mortality. The parameters measure two dimensions: the level and the pattern of mortality. The model is applied to real and model life tables and compared with Brass's logit model. The model is shown to fit as well as the Brass model without the difficulty of choosing a standard table.

Résumé

Un modèle de mortalité à deux paramètres est présenté—la simplification d'un modèle à trois paramètres formulé par Mitra (1983). Il exprime $\ell n(-\ell(x))$ comme une fonction linéaire de $\ell n(x)$ et $\ell n(\alpha - x)$ où x est l'âge, α est la limite d'âge supérieure et $\ell(x)$ est la probabilité de survie de la naissance à l'âge x. Cette simplification est réalisée en forçant le modèle à reproduire la valeur donnée de la mortalité infantile. Les paramètres mesurent deux dimensions : le niveau et la configuration de la mortalité. Le modèle est appliqué aux tables de mortalité réelle et modèle, et comparé au modèle logit de Brass. Il s'avère que ce modèle est aussi bien ajusté que le modèle Brass sans imposer la difficulté de choisir un tableau standard.

Key Words: weighted linear regression with constraint, survivorship function, infant mortality, force of mortality

The Problem

The practical need for models in many fields of scientific endeavor has often spurred on investigations where the end results are substantiated more by empirical tests rather than by strong theoretical arguments. In the field of demography, this is most exemplified by the history of modeling life table functions which perhaps began with Gompertz (1825). He attempted to describe the force of mortality as an exponential function of age by arguing that man's ability to resist death decreases with age. The fit was found to be reasonably satisfactory except for the extreme ages and especially so for the childhood age range where mortality declines rather than increases with age. Makeham (1860) inserted an additive constant to the Gompertz function by noting that a chance factor independent of age also affects mortality. His three parameter model produced a better fit but still left much to be desired.

Many years later, Perks (1932) modified the Makeham formula by incorporating a denominator comprised of a linear compound of two exponential functions, increasing the number of parameters to five in the process. That did not stop the search for a better fit and of late, Heligman and Pollárd (1980) came up with an eight parameter model to fit the curve generated by the probability of dying at successive ages. An excellent summary and discussion of the utility of these models may be seen in a recent paper by Keyfitz (1991).

Petrioli and Berti (1979) have proposed an interesting model by first defining a mortality resistance function

$$r(x) = \frac{\ell(x)}{1-\ell(x)} \cdot \frac{x}{\alpha - x} \tag{1}$$

which is zero at both ends of the age continuum and positive elsewhere. Its pattern of distribution has been empirically demonstrated to be a bell-shaped curve; the functional form of which could be expressed as

$$r(x) = x^{E} (\alpha - x)^{F} e^{ux^2+vx+w} \tag{2}$$

Substitution of (2) in (1) results in an equation that expresses $\ell(x)$ as a five parameter function of x. The parameters have been estimated by solving (2) for selected values of x which includes the age at which r(x) assumes its maximum value. In spite of their claim for a good fit, the number of parameters, the nonlinearity of the model, the functional form of r(x), and the nature of variation of the parameters over levels of mortality, leave much to be desired. Besides, given five points, a fourth degree polynomial would do just as well or better as Valcovics (1993) has found with a variety of life tables.

Another attempt to model mortality has been made by Gavrilov and Gavrilova (1991) who have proposed an "avalanche-like destruction" of an organism in natural aging. The model they came up with after defining a number of parameters can be written in a simpler form as

$$\ell n \ell(x) = A + Bx + C \ell n(1 + De^{-Ex}) \qquad (3)$$

Unfortunately, the novelty of the idea did not materialize in reality and the authors are hopeful that the model can be improved by further research. If additional parameters beyond its present five are needed to produce a better fit, then the model's usefulness will be severely reduced.

An indirect approach to modeling life table functions by Brass (1978) produced a linear relationship between the logits of the survivorship functions of two life tables. The goodness of fit with this model in any given exercise depends on the selection of a standard life table. It has been found that the procedure has a tendency to overestimate life expectancy by overestimating $\ell(x)$ at older ages (Keyfitz, 1991). However, due to the simplicity of the model (for its dependence on only two parameters), users overlook the deficiencies. Thus, although the Brass model cannot be regarded as one that is a direct function of age, we have taken the liberty of comparing its goodness of fit with our model presented below.

Some years ago, Mitra (1983, 1984) experimented with a mortality model which was developed from the observed pattern of the distribution of the force of mortality. A simple formulation of the functional relationship between a person's ability to withstand death and the person's age, led to the model expressing $\ell n(-\ell n\ \ell(x))$ as a linear function of ℓnx and $\ell n(\alpha - x)$, where x is age, α is the upper limit of life, and $\ell(x)$ is the probability of surviving from birth to age x. That model produced encouraging results when tested against both Coale and Demeny's (1983) regional model as well as real life tables.

One of the ways that this three parameter linear model can be used is to generate model life tables by letting the parameters vary. However, the usefulness and the validity of the model depend also on its ability to generate data from limited information. The purpose of this paper is to investigate whether the model can generate consistent life table functions from given or estimated value(s) of $\ell(x)$ at a critical point, specifically, at age one.

The Model and the Method of Estimating its Parameters

Let us begin with a brief overview of the derivation of the functional form of the proposed model, henceforth referred to as the double log model. Denoting by $\ell(x)$, the life table survivorship function at age x and

$$\mu(x) = -\frac{d\ell n\ell(x)}{dx} \tag{4}$$

the force of mortality at age x, one can write

$$\ell(x) = e^{-f(x)}, \tag{5}$$

where
$$f(x) = \int_0^x \mu(x)\,dx. \tag{6}$$

Observe that f(x) must meet the following conditions

(a) $f(0) = 0$ since $\ell(0) = 1$
(b) $f(\alpha) = \infty$ so that $\ell(\alpha) = 0$, where α is the life span
(c) $f'(x)$ or $\mu(x)$ is uniformly positive since $\ell(x)$ is a monotonically nonincreasing function of age. Also $\mu(x)$ assumes very large values at two extremes and a minimum value at an appropriate age.

Clearly, a simple algebraic function that is proportional to x^m meets (a) with

$$0 < m, \tag{7}$$

and another function that is proportional to the reciprocal of $(\alpha\text{-}x)^n$ meets condition (b) with

$$n > 0. \tag{8}$$

Combining the two as

$$f(x) = \frac{Ax^m}{(\alpha-x)^n} \qquad \text{with } A > 0 \tag{9}$$

we get a function which meets conditions (a) and (b). Interestingly enough, its derivative

$$f'(x) = \mu(x) = \frac{mAx^{m-1}}{(\alpha-x)^n} + \frac{nAx^m}{(\alpha-x)^{n+1}} \tag{10}$$

can be made to meet condition (c) by setting a limit on the permissible range of variation for the parameter m. Note from (10) that

$$m \not> 1 \tag{11}$$

since $\mu(x)$ is large at age 0. Therefore, combining with (7), the boundary condition for m can be specified as

$$0 < m \leq 1. \tag{12}$$

It has been shown (Mitra, 1984) that the empirical fact that the force of mortality assumes its minimum value at a relatively early age, requires

$$n > m. \tag{13}$$

The model described in (5) and (9) can be linearized by applying the logarithmic transformation twice to get

$$\ell n(-\ell n\, \ell(x)) = \ell n\, A + m\, \ell n x - n\, \ell n(\alpha - x). \tag{14}$$

The parameters have been previously estimated by Mitra (1984) for selected life tables by using the straightforward method of least squares while setting a constant value of α such as 100 or 110.

The usefulness of the model can be further demonstrated by showing that it can be reduced to a two parameter model for given values of α, and a single piece of a priori information such as the infant mortality rate. Since the complement of infant mortality rate is $\ell(1)$, it follows that for the model to reproduce the infant mortality, the identity

$$\ell n(-\ell n \ell(1)) = \ell n A + 0 - n\, \ell n(\alpha - 1) \tag{15}$$

must hold. Subtraction of (15) from (14) then gives

$$\ell n(-\ell n \ell(x)) - \ell n(-\ell n \ell(1)) = m\, \ell n x - n[\ell n(\alpha - x) - \ell n(\alpha - 1)] \tag{16}$$

the right hand side of which, for a given value of α, is a linear function of $\ell n x$ and $\ell n(\alpha - x) - \ell n(\alpha - 1)$ with two parameters, m and n. For the estimation of these parameters, (16) can be treated as a linear regression model with zero intercept. However, it may be noted that the dependent variable does not have a constant variance since for a given value of x, the variance of $\ell(x)$ can be estimated as

$$V(\ell(x)) = \ell(x)(1-\ell(x)). \tag{17}$$

As the variance of a function of y, f(y) is approximated by the formula

$$V(f(y)) \approx V(y)\,[f'(y)]^2_{y=E(y)} \tag{18}$$

where E(y) is the expected value of y, we can derive the variance of the dependent variable for a given value of $\ell(1)$ as

$$\begin{aligned} V[\ell n(-\ell n\ell(x)) - \ell n(-\ell n\ell(1))] &= \frac{\ell(x)\,(1-\ell(x))}{\ell^2(x)\,(\ell n\ell(x))^2} \\ &= \frac{1-\ell(x)}{\ell(x)\,(\ell n\ell(x))^2} \end{aligned} \tag{19}$$

resulting from equations (17) and (18). Consequently, the parameters of the linear model (16) can be better estimated by the method of weighted least squares with the constraint that the intercept is zero. For a given x, the weight is the corresponding value of the reciprocal of (19).

It may be seen that when the constants are estimated by following this procedure, (16) can be expressed in the form of (14) by writing the intercept term as

$$\ell n A = \ell n(-\ell n\ell(1)) + n\,\ell n(\alpha-1) \tag{20}$$

Experiment with the Double Log Model and Discussion of Results

Two data sets were employed to examine the double log model: Coale and Demeny's (1983) model life tables and real life tables from the 1985 United Nations Demographic Yearbook (1985). A set of 52 life tables were selected from Coale and Demeny's (1983) tables covering the entire range of female life expectancy from 20 to 80 years, with the difference between the life expectancies of two successive life tables held constant at 5 years. Male life tables were chosen which corresponded to those female tables with the 5 year interval between life expectancies. The 52 life tables comprised four subsets of 13 life tables selected from each of the four regions of Coale and Demeny's (1983) model life tables. For operational convenience, female tables were selected from East and West and male tables from North and South.

A sample of six life tables covering a wide range of mortality levels were selected for study from the 1985 United Nations Yearbook (1985). These tables were Botswana 1980-81 male and female, e(0) = 52.72 and 59.32, Bahrain 1976-81 male and female, e(0) = 63.32 and 66.25, and Japan 1984 male and female e(0) = 74.41 and 79.75. Furthermore, four other United Nations (1985) life tables were chosen as standard life tables for the Brass model. Two "low" mortality tables, England and Wales 1982-84 male and female, e(0) = 71.48 and 77.04, and two "high" mortality tables Mexico 1970 male and female, e(0) = 59.14 and 63.05, were selected.

Estimates of the parameters "m" and "n" of equation (16) were found for each life table using the method of weighted least squares with the reciprocal of (19) as the weight. The value of α was set at 100. Since $\ell n(\alpha-x)$, an element of our regression equation (16), is undefined at age 100, the 20 ages used in the computations ranged from 1 to 95 for the model life tables. Because the highest age level for the United Nations (1985) life tables is 85 years, only 18 ages were used in those computations. The results of these regressions are summarized in Tables 1 and 2 for the model tables and Table 3 for the six national tables. For the model tables, each life table is identified by its geographical region, a level number assigned by Coale and Demeny (1983), and its life expectancy. The values of $\ell(1)$ and the square of the multiple correlation coefficient, R^2, are presented for each life table.

Note that the values of "m" and "n" meet the boundary conditions set in (12) and (13) with one minor exception in the South level 23 for the male life table. The value of -.001 of the parameter m is not significantly different from 0 and accordingly, the derivation of expected values of life tables should be based on m = 0 for consistency. Next, the readers may like to note that the parameter "n" uniformly increases with decline in mortality or in life expectancy. The parameter "m" on the other hand shows an inverse relationship, i.e., it decreases with life expectancy until level 19 or a life expectancy of 65 is reached in both the East and West region. The pattern is the same for North region males, while for the South region males the decline continues to the next to the last level. This phenomenon can perhaps be explained by the behavior of the parameter "m" which can be decomposed into two components (Mitra, 1983). The first contributes to a decline in mortality with increase in age, while the second keeps the mortality increasing with age. The combined effect of these two components for various levels of mortality seems to be the cause of the reversal of its trend. The contribution of "n" on the pattern of mortality, however, is more straightforward as it varies directly with life expectancy. Thus, "m" is more a measure of the overall pattern of mortality, while "n" is an indicator of the overall level of mortality. From a technical point of view, an infinite number of life tables can be constructed by combining any value of "m" with any value of "n" as long as they meet the boundary conditions. In the regional model tables of Coale and Demeny (1983), the region may be looked upon as a measure of the pattern of mortality, and the life expectancy as the overall measure of mortality. It may therefore be said, that as those life tables are based on two independent dimensions of mortality, the double logarithmic model presented in this paper also has two, albeit different, dimensions.

TABLE 1. THE PARAMETERS OF THE REGRESSION EQUATION (16) FOR SELECTED FEMALE LIFE TABLES.

Region	*Level*	*e(0)*	*ℓ(1)*	*m*	*n*	R^2
East	1	20	.57180	.128**	.899**	.99524
	3	25	.63788	.122**	.967**	.99580
	5	30	.69350	.116**	1.028**	.99628
	7	35	.74135	.108**	1.088**	.99666
	9	40	.78317	.100**	1.150**	.99693
	11	45	.82003	.089**	1.223**	.99713
	13	50	.85260	.073**	1.318**	.99709
	15	55	.88267	.057**	1.417**	.99677
	17	60	.91028	.043*	1.523**	.99607
	19	65	.93548	.030	1.646**	.99481
	21	70	.95904	.033	1.776**	.99293
	23	75	.97861	.087	1.838**	.99164
	25	80	.99245	.245**	1.812**	.99239
West	1	20	.63445	.192**	.902**	.99836
	3	25	.69444	.188**	.950**	.99867
	5	30	.74389	.184**	.995**	.99885
	7	35	.78571	.179**	1.042**	.99892
	9	40	.82178	.173**	1.095**	.99891
	11	45	.85336	.165**	1.155**	.99881
	13	50	.88121	.156**	1.226**	.99857
	15	55	.90606	.140**	1.328**	.99814
	17	60	.92884	.127**	1.439**	.99732
	19	65	.94965	.117**	1.564**	.99604
	21	70	.96884	.123**	1.707**	.99412
	23	75	.98470	.187**	1.786**	.99277
	25	80	.99555	.397**	1.752**	.99376

*p<.05 **p<.01

TABLE 2. THE PARAMETERS OF THE REGRESSION EQUATION (16) FOR SELECTED MALE LIFE TABLES.

Region	*Level*	*e(0)*	*l(1)*	*m*	*n*	R^2
North	1	17.551	.62858	.229**	.825**	.99671
	3	22.341	.68919	.223**	.891**	.99732
	5	27.163	.73883	.218**	.946**	.99781
	7	32.012	.78062	.213**	.994**	.99820
	9	36.884	.81650	.208**	1.040**	.99847
	11	41.779	.84777	.203**	1.087**	.99862
	13	46.697	.87529	.196**	1.145**	.99870
	15	51.440	.89858	.185**	1.222**	.99857
	17	56.319	.92054	.177**	1.299**	.99819
	19	61.312	.94091	.173**	1.382**	.99752
	21	66.391	.95950	.175**	1.480**	.99645
	23	71.585	.97580	.196**	1.581**	.99497
	25	77.289	.98944	.281**	1.627**	.99447
South	1	19.920	.66423	.245**	.742**	.99375
	3	24.661	.71056	.227**	.835**	.99448
	5	29.337	.74894	.209**	.916**	.99512
	7	33.950	.78154	.191**	.992**	.99568
	9	38.501	.80975	.172**	1.068**	.99614
	11	42.861	.83337	.150**	1.152**	.99657
	13	47.372	.85474	.122**	1.244**	.99679
	15	51.869	.87498	.094**	1.338**	.99675
	17	56.341	.89397	.064**	1.437**	.99636
	19	61.252	.91361	.028	1.554**	.99528
	21	66.080	.93372	.007	1.642**	.99375
	23	70.993	.95343	-.001	1.716**	.99199
	25	76.002	.97178	.019	1.765**	.99076

*p<.05 **p<.01

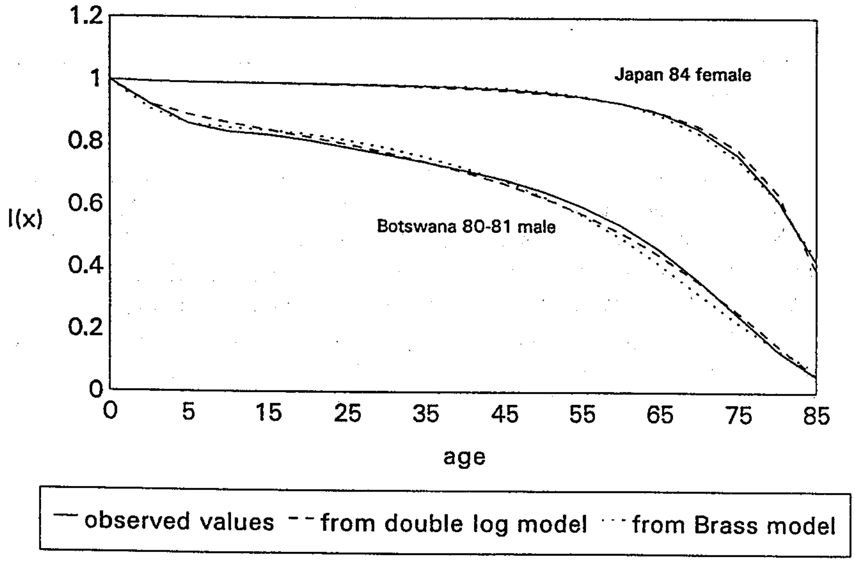
The goodness of fit of the model was measured in two ways. First, examination of the square of the multiple correlation coefficient, R^2, in Tables 1 and 2 shows a range from .99164 to .99963 for both the model and real life tables. Second, encouraging results were obtained when the actual values of $\ell(x)$ were regressed on the expected values of $\ell(x)$ using ordinary least squares. A perfect model would produce an intercept of zero and a slope of one. For the 52 model life tables and the six national tables, the largest absolute value of the intercept was .051 and the slope ranged from .923 to 1.073.

TABLE 3. THE PARAMETERS OF THE REGRESSION EQUATION (16) FOR SIX NATIONAL LIFE TABLES (B6 ASCENDING LIFE EXPENTANCY AT BIRTH.)

Sex/ Country	year	e(0)	l(1)	m	n	R^2
Male						
Botswana	80-81	52.72	.92402	.207**	1.419**	.99906
Bahrain	76-81	63.32	.94669	.039*	1.918**	.99873
Japan	84	74.41	.99338	.200**	2.449**	.99912
Female						
Botswana	80-81	59.32	.93941	.153**	1.546**	.99818
Bahrain	76-81	66.25	.95130	.036*	1.836**	.99910
Japan	84	79.75	.99471	.071**	2.571**	.99963

*p<.05 **p<.01

FIGURE 1. OBSERVED AND EXPECTED $\ell(x)$ VALUES DERIVED FROM THE DOUBLE LOG AND BRASS MODELS FOR TWO LIFE TABLES



The standard table for Botswana is Mexico 70 male. For Japan it is England and Wales 82-84 female.

TABLE 4. RESULTS OF REGRESSING ACTUAL $\ell(x)$ FOR THE DOUBLE LOGARITHM MODEL AND THE BRASS MODEL, USING BOTH ENGLAND AND WALES 82-84 (BRASS-E&W) AND MEXICO 70 (BRASS-MEX) AS STANDARD TABLES.

Country	*Year*	*Sex*	*Model*	*Slope*	*Intercept*	R^2
Botswana	80-81	male	double log	.985	.008	.99630
			Brass-E&W	.952	.032	.98557
			Brass-Mex	.966	.025	.99271
		female	double log	.985	.009	.99332
			Brass-E&W	.962	.032	.97599
			Brass-Mex	.977	.020	.99314
Bahrain	76-81	male	double log	1.011	-.008	.99537
			Brass-E&W	.998	.005	.99641
			Brass-Mex	1.015	-.008	.99781
		female	double log	1.003	-.003	.99679
			Brass-E&W	1.007	-.002	.99046
			Brass-Mex	1.011	-.007	.99923
Japan	84	male	double log	1.001	.000	.99340
			Brass-E&W	.959	.034	.99864
			Brass-Mex	.964	.030	.99773
		female	double log	.991	.008	.99624
			Brass-E&W	1.004	-.003	.99830
			Brass-Mex	.949	.047	.99818

Another way in which the double log model was evaluated was to compare it with Brass's logit model. For the six national life tables, expected values of $\ell(x)$ were generated using this model and the Brass model. Expected values for the Brass model were calculated using both "low" and "high" mortality standard life tables, England and Wales 1982-84 and Mexico 1970. The observed $\ell(x)$ values were regressed on the expected values and the resulting slopes, intercepts, and values of R^2 are presented in Table 4. Figure 1 graphically compares two life tables with the observed values of $\ell(x)$ and the expected values from both the double log model and the Brass model, using the standard table which produced the best results. The values of $\ell(x)$ on which Figure 1 is based, can be found in Table 5. Examination of Tables 4 and 5, and Figure 1, show that the double log model fits as well as Brass's model. The main advantage of the double log model over the Brass model is that a standard table need not be chosen.

TABLE 5. ACTUAL AND EXPECTED $\ell(x)$ VALUES DERIVED FROM THE DOUBLE LOG MODEL AND THE BRASS MODEL

	Botswana 80-81 Male			*Japan 84 Female*		
Age	Actual	Double Log Model	Brass-Mex Model	Actual	Double Log Model	Brass-E&W Model
1	.92402	.92402	.90803	.99471	.99471	.99342
5	.86132	.88973	.86132	.99280	.99341	.99230
10	.83603	.86452	.84867	.99200	.99205	.99165
15	.82603	.84225	.84097	.99137	.99053	.99099
20	.80918	.81991	.82863	.99017	.98871	.98993
25	.78660	.79623	.80966	.98844	.98647	.98879
30	.76362	.77032	.78580	.98630	.98366	.98730
35	.73997	.74136	.75689	.98354	.98005	.98462
40	.71371	.70848	.72177	.97949	.97532	.98210
45	.68337	.67064	.67999	.97339	.96897	.97695
50	.64607	.62661	.63049	.96407	.96024	.96796
55	.59844	.57488	.57007	.95020	.94785	.95297
60	.53808	.51368	.49712	.93013	.92965	.92861
65	.45830	.44108	.41149	.89898	.90175	.89102
70	.36130	.35546	.31782	.84794	.85682	.83456
75	.24741	.25694	.22345	.76303	.78024	.74827
80	.13529	.15106	.13905	.62155	.64247	.61810
85	.05274	.05622	.07150	.42027	.39417	.43436

The standard table for Botswana is Mexico 70 male (Brass-Mex). For Japan it is England and Wales 82-84 female (Brass-E&W).

This model can be used to generate the values of the survivorship function given only the infant mortality, $1-\ell(1)$, and the region in which the country is located. For example, if $\ell(1)$ is assigned a value of say, .97677 for females in a country in the West region, one can interpolate to find values of "m" and "n" from the bottom half of Table 1. Since .97677 is halfway between the $\ell(1)$ values for female West levels 21 and 23, the technique of linear interpolation then produces estimates of the parameters "m" and "n" as m=.155 and n=1.747. To find $\ell(20)$, for instance, one simply substitutes the values of $\ell(1)$, m, and n along with the value of 100 for α and 20 for x into equation (16). The solution for $\ell(20)$ in this illustration is .94719. In like manner, one can generate values of the survivorship function for other ages. From the $\ell(x)$ values, other life table functions can be determined. However, the application of the model is by no means restricted to the choice of one of the regions of Coale and Demeny's (1983) model tables. As may be seen from Tables 1 and 2, a region corresponds to specific sequences of values of "m" and "n" for each sex. From a theoretical point of view, an infinite number of such sequences can be created subject to the boundary conditions (12) and (13), each capable of generating a separate series of life tables.

Concluding Remarks

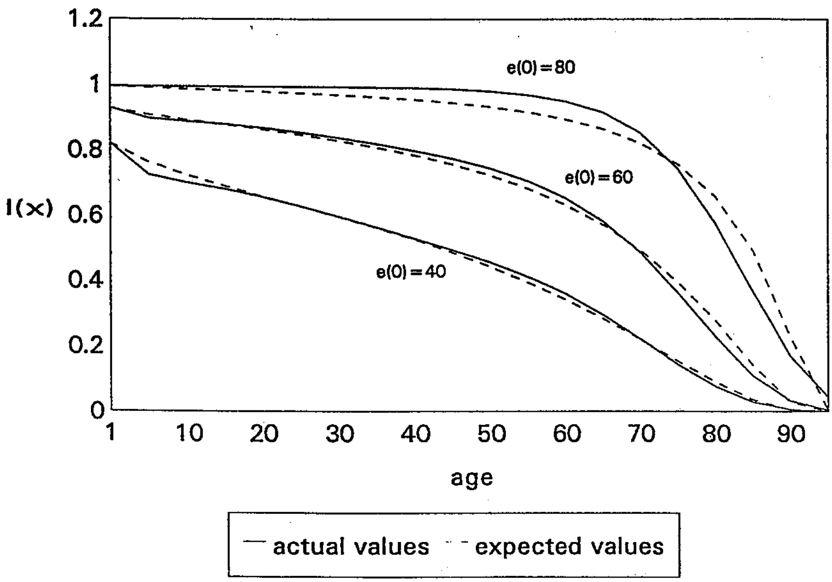
The double log model produces consistent life table functions from a given or estimated value(s) of $\ell(x)$ at age one. With our sample of national life tables, the double log model produced expected values of $\ell(x)$ as close to the observed values as the Brass model. The regressions of actual $\ell(x)$ on expected $\ell(x)$ using the model life tables also demonstrated the strength of the double log model. The great advantage of the double log model over the commonly used Brass model is that its use does not require the selection of a standard table.

TABLE 6. ACTUAL AND EXPECTED $\ell(x)$ VALUES FROM THE DOUBLE LOG MODEL FOR SELECTED FEMALE. WEST LIFE TABLES.

	e(0) = 40		e(0) = 60		e(0) = 80	
age	act.	exp.	act.	exp.	act.	exp.
1	.82178	.82178	.92884	.92884	.99555	.99555
5	.72459	.76239	.89772	.90842	.99522	.99096
10	.70001	.72288	.88778	.89283	.99499	.98694
15	.68147	.69033	.88013	.87848	.99478	.98308
20	.65758	.65953	.86906	.86362	.99440	.97896
25	.62856	.62859	.85460	.84747	.99384	.97431
30	.59737	.59646	.83819	.82941	.99306	.96893
35	.56387	.56239	.81969	.80881	.99198	.96251
40	.52894	.52574	.79867	.78493	.99022	.95470
45	.49323	.48589	.77437	.75685	.98697	.94500
50	.45631	.44225	.74489	.72337	.98036	.93266
55	.41175	.39422	.70574	.68289	.96904	.91661
60	.35989	.34130	.65424	.63326	.94948	.89512
65	.29435	.28322	.58249	.57160	.91660	.86544
70	.22191	.22031	.48711	.49421	.85479	.82285
75	.14442	.15427	.36469	.39683	.74670	.75895
80	.07544	.08955	.22853	.27674	.57547	.65813
85	.02811	.03540	.10829	.14108	.36683	.49194
90	.00603	.00520	.03250	.02915	.16927	.22841
95	.00055	.00001	.00467	.00006	.04484	.00621

Interesting common patterns were revealed when studying the graphs of the actual and expected $\ell(x)$ values for both sexes. The patterns are evident in the actual and expected values of $\ell(x)$ presented for three female West life tables in Figure 2 and Table 6. Since both the given and the expected $\ell(x)$ functions are monotonic nonincreasing functions of age as they should be, they are expected to crisscross unless the fit is perfect. In fact, crisscrossing is the next best thing after the ideal case of a perfect fit which cannot be expected from any model. Crisscrossing is the only way the desirable condition of nearly identical grand totals of the two distributions can be met.

FIGURE 2. ACTUAL AND EXPECTED $\ell(x)$ VALUES FOR SELECTED FEMALE WEST MODEL LIFE TABLES.



For both males and females, the crisscrossing phenomenon begins with producing overestimates at younger ages in the high mortality example. The magnitude of the overestimate decreases with declining mortality and finally, the pattern is reversed at the other extreme. It may be mentioned at this point that we are comparing our model tables with another set of tables which are also model tables themselves. The adequacy of the latter tables was never tested by comparing those with the life tables selected to generate them. As a matter of fact, for these regional life tables, no mechanism seems to exist which can be used to identify a model counterpart of an actual life table. Those tables, like ours, meet certain basic requirements such as decreasing mortality from birth to a certain age, increasing mortality past that age, monotonic nonincreasing nature of the $\ell(x)$ function. These conditions can be met by an innumerable number of life tables, and our own double logarithmic model is also capable of producing an infinite number of such tables. From that perspective, the regional tables are somewhat limited with only four regions.

As noted in the previous section, the double logarithmic model tables, like the regional tables, have two dimensions. It is possible, that future investigations along this line, may further improve upon the goodness of fit by developing another method for defining the dimensions, or by adding an additional dimension to it. The latter approach will necessitate the inclusion of another parameter in the model, which from the point of view of simplicity and parsimony, should only be considered as the last alternative.

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