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Experiments in the Projection of Mortality

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Résumé
Une des façons recommandées de projeter la mortalité est de paramétrer les séries de tables de mortalité accessibles et d'extrapoler chacun des paramètres séparément en une série chronologique afin de dégager les tendances futures. Les courbes paramétriques sont comparées et, en général, un plus grand nombre de paramètres sont plus représentatifs des tables individuelles; par ailleurs, un nombre plus élevé de paramètres donne des chronogrammes moins réguliers, qui se prêtent plus difficilement à une extrapolation dans l'avenir. On ne peut toutefois s'abstenir de prédire à quelle période passée, ou moyenne de périodes passées, ressemblera l'amélioration future de la mortalité.

Abstract
A recommended way of projecting mortality has been parameterizing the series of life tables available, extrapolating each parameter separately as a time series, and so constructing the future. Curves for parameterizing are compared and, in general, more parameters give better fits to individual tables, but more parameters turn out to have less regular time series, and are therefore more difficult to extrapolate into the future. There can be no avoiding the decision on what past period or average of past periods the future improvement of mortality is likely to resemble.

Key Words — life tables, future mortality, extrapolation, curve fitting

How fast will mortality fall in the decades ahead? One way of phrasing the question is in terms of past periods: will it be as fast as Canada showed in 1976-81, or only as fast as the average 1921-81, or as slow as 1926-31? This apparently simple question — asked and answered in perfectly non-technical language — will be shown to deal with the question of future mortality as well as more sophisticated methods. I will argue that the whole matter of projecting mortality comes down to deciding what past period describes the future.

One could think of the mortality fall of the several five-year time intervals as a random variable. Then each of the 12 five-year intervals from 1921 to 1981 would be seen as providing a random value of the variable "mortality fall." Between the extremes of the past — the small improvement of 1926-31 and the large improvement of 1976-81 — is presumably the range within which some large fraction of the probability for the future lies. More refined methods for dealing with error are given by Keilman and Kucera (1989).

Extrapolating Survivorship or Death?

One starts with geometric extrapolation using the minimum of data, first finding the ratio of the q_x of 1981 to that of 1976 at each separate age, and taking this as the ratio for all times in the future. The result is compared with the same geometric extrapolation, but on the complement of survivorship, $1 - l_x$, where again the ratio of improvement is taken from the last time interval, 1976-81 (Table 1). Table 2 takes the ratio of improvement from the last six time intervals, 1951 to 1981.

TABLE 1. LIFE EXPECTANCY $\mathring{e}_0$ FOR 1986-2021 ON TWO METHODS, SHOWING THE EFFECT ON OF $\mathring{e}_0$ GEOMETRIC EXTRAPOLATION ON q_x VERSUS GEOMETRIC EXTRAPOLATION ON THE COMPLEMENT OF SURVIVORSHIP, $1 - l_x$, WHERE THE RATIO OF IMPROVEMENT IS TAKEN FROM THE TIME INTERVAL, 1976-81

	1986	1991	1996	2001	2006	2011	2016	2021
q_x	76.805	77.960	78.999	79.937	80.786	81.556	82.256	81.894
$1-l_x$	76.575	77.525	78.383	79.160	79.866	80.510	81.100	81.641

The geometric series based on q_x gives higher survivorship, with 1.25 years more by the year 2021. Both of these are high compared with what we will see below, and that is due to the ratio used — 1976 to 1981 — being the largest improvement of mortality in the 60 year record.

Table 3 includes a third method devised by Brass (1975). It will be recalled that the Brass method consists in first transforming the l_x to logits, say Y_x, where

$$Y_x = \frac{1}{2} \log \left(\frac{1 - l_x}{l_x} \right)$$

then choosing one of the life tables (in our case the most recent) as the standard, then finding the simple regression of each of the other tables on the standard, so obtaining an alpha and a beta, intercept and slope, for each life table. Each of alpha and beta forms a time series, and the two time series may be projected — in our case with a straight line fitted by least squares. As among the three different ways of projecting mortality, geometric projection of q_x; geometric projection of the complement of l_x, $1-l_x$; and the Brass procedure, the last named gives the highest expectancy. Table 4 shows forecasts for l_1, l_{50} and l_{85} on all three methods. The differences between the three become considerable only at very old ages.

TABLE 2. SAME AS TABLE 1 GIVING $\mathring{e}_0$ BUT BASING THE RATIO FOR THE FUTURE ON THE AVERAGE IMPROVEMENT OF THE LAST SIX INTERVALS, THAT IS, ON THE AVERAGE OF 1951-81

	1986	1991	1996	2001	2006	2011	2016	2021
q_x	76.287	77.002	77.667	78.289	78.873	79.423	79.941	80.432
$1-l_x$	76.156	76.750	77.305	78.825	78.312	78.770	79.201	79.608

TABLE 3. SAME AS TABLE 1 GIVING $\mathring{e}_0$ B UT BASING THE IMPROVEMENT ON THE AVERAGE IMPROVEMENT OF ALL 12 INTERVALS, THAT IS, ON THE AVERAGE OF 1921-81, PROJECTING WITH q_xAND $1-l_x$, AND ALSO USING THE BRASS METHOD

	1986	1991	1996	2001	2006	2011	2016	2021
q_x	76.162	76.759	77.315	77.832	78.317	78.772	79.200	79.604
$1-l_x$	76.048	76.541	77.002	77.433	77.838	78.218	78.577	78.915
Brass	76.019	76.925	77.775	78.573	79.322	80.025	80.686	81.306

TABLE 4. VALUES OF l_1, l_{50} aAND l_{85} PROJECTING WITH q_x AND BRASS, INCREASE OF ALL 12 INTERVALS

	1986	1991	1996	2001	2006	2011	2016	2021
Values of l_1								
q_x	0.9921	0.9934	0.9945	0.9954	0.9961	0.9968	0.9973	0.9977
$1-l_x$	0.9921	0.9934	0.9946	0.9955	0.9962	0.9969	0.9974	0.9979
Brass	0.9917	0.9932	0.9943	0.9953	0.9961	0.9968	0.9973	0.9978
Values of l_{50}								
q_x	0.9392	0.9451	0.9503	0.9550	0.9592	0.9629	0.9662	0.9692
$1-l_x$	0.9387	0.9443	0.9494	0.9540	0.9582	0.9620	0.9655	0.9686
Brass	0.9385	0.9465	0.9536	0.9598	0.9652	0.9699	0.9739	0.9775
Values of l_{85}								
q_x	0.3161	0.3311	0.3460	0.3607	0.3753	0.3898	0.4041	0.4182
$1-l_x$	0.3082	0.3154	0.3224	0.3294	0.3363	0.3432	0.3500	0.3567
Brass	0.3055	0.3227	0.3405	0.3587	0.3774	0.3964	0.4157	0.4353

The summary in Table 5 demonstrates that which of the past sets of data used matters more than the method used to extrapolate from the data. Taking just the last interval and the whole 12 intervals, for $\overset{\circ}{e}_0$

Extrapolating on	last interval	all 12 intervals	difference due to choice of interval
q_x	82.894	79.604	3.290
$1-l_x$	81.641	78.915	2.726
difference due to method	1.253	0.689	

TABLE 5. SUMMARY FOR THE YEAR 2021, PROJECTING WITH q_x, l_x AND BRASS, AND USING THREE PAST PERIODS FOR DATA

	e_0	l_1	l_{50}	l_{85}
On Increase of Last Interval				
q_x	82.894	0.9991	0.9827	0.5680
$1\text{-}l_x$	81.641	0.9991	0.9811	0.4642
On Average Increase of Last 6 Intervals - 1951-1981				
q_x	80.432	0.9985	0.9664	0.4769
$1\text{-}l_x$	79.608	0.9984	0.9681	0.3916
On Average Increase of all 12 Intervals - 1921-1981				
q_x	79.604	0.9977	0.9692	0.4182
$1\text{-}l_x$	78.915	0.9979	0.9686	0.3567
Brass	81.306	0.9978	0.9775	0.4353

Note again that the Brass method shows higher survivorship, and accordingly higher life expectancies and higher projected population, than either of the other two projections, when all three are applied to the same set of data — that is, the 13 Canadian life tables. Since all methods are biased downwards by truncation of the table at age 90, this is an advantage of the Brass approach. Even so, the difference is no more important than the difference among past periods chosen as describing the future.

Comparison with the United Nations Procedure

Let us compare all these with the United Nations (1989) projections (Table 6). Apparently the U.N. $\overset{\circ}{e}_0$ is lower than the result of projecting with the mortality of the late 1970s, and higher than the use of all 13 life tables by geometric series. As among the three past periods chosen as base, it most nearly coincides with what we obtained by using the average ratio of improvement of the last seven life tables, that is of the interval 1951-81.

TABLE 6. COMPARISON OF THE THREE TIME PERIODS OF TABLES 1 THROUGH 4, USING q_x IN GEOMETRIC PROGRESSION, AND THE BRASS METHOD

	1976-81 1)	1951-81 2)	1921-81 3)	Brass 4)	UN 5)	Departures from UN 1)-5)	2)-5)	3)-5)	4)-5)
1981	75.51	75.51	75.51	75.51	75.92	-0.41	-0.41	-0.41	-0.41
1986	76.80	76.28	76.16	76.02	76.69	0.11	-0.41	-0.54	-0.68
1991	77.96	77.00	76.76	76.92	77.29	0.67	-0-.29	-0.54	-0.37
1996	79.00	77.67	77.31	77.77	77.97	1.03	-0.30	-0.65	-0.19
2001	79.94	78.29	77.83	78.57	78.51	1.42	-0.23	-0.68	0.06
2006	80.79	78.87	78.32	79.32	79.03	1.76	-0.16	-0.71	0.29
2011	81.56	79.42	78.77	80.02	79.48	2.07	-0.06	-0.71	0.54
2016	82.26	79.94	79.20	80.69	79.95	2.30	-0.01	-0.75	0.73
2021	82.89	80.43	79.60	81.31	80.41	2.48	0.02	-0.81	0.89

Aside from checking the author's method against that of the U.N., the comparison in Table 6 serves to evaluate the U.N. method, to see what implicit assumption underlies it. Readers are not informed what method the U.N. actually used, but its outcome after the year 2000 is almost exactly equivalent to projecting q_x in geometric progression, using the average ratio of 1951 to 1981. Since it seems likely that the future can show more progress than the future can show more progress than the 1921-81 average, but probably not as much as 1976-81, this intermediate period seems to generate the most reasonable estimate.

Effect on the Projected Population

With each one of the mortality extrapolations considered, a full population projection is made in order to see what is the corresponding future population, using some standard set of the fertility and migration components. Table 7 compares the consequences for the output population when a given set of data is used, and different methods employed. In all cases, as Table 7 shows, the use of q_x in a simple geometric ratio projection estimates a larger population than does the projection of $1-l_x$, and again the Brass method is higher than either one. However, the choice of the period from which the ratio is selected is again, on the whole, more important that the choice of method.

TABLE 7. PROJECTED POPULATION 1986-2021 ON THREE SETS OF DATA, SHOWING THE EFFECT OF DIFFERENT METHODS FOR ANY ONE SET (THOUSANDS OF PERSONS)

	1986	1991	1996	2001	2006	2011	2016	2021
On Increase of Last Interval								
q_x	24089	25593	27101	28433	29619	30657	31644	32639
$1-l_x$	24089	25579	27059	28352	29492	30477	31410	32350
On Average Increase of Last 6 Intervals								
q_x	24089	25566	27023	28282	29378	30310	31181	32048
$1-l_x$	24089	25557	26996	28229	29295	30194	31031	31865
On Average Increase of all 12 Intervals								
q_x	24089	25559	27000	28238	29308	30210	31046	31874
$1-l_x$	24089	25550	26976	28192	29236	30110	30920	31723
Brass	24089	25550	27001	28269	29390	30364	31295	32242

For example, for the year 2021 one sees from Table 7, in thousands of persons,

	last interval	all 12 intervals	difference
q_x	32639	31874	765
$1-l_x$	32350	31723	627
difference	289	151	

This is similar to the effect on $\mathring{e}_0$ where, as already noted, the base time period matters some three times as much as the method.

Curve Fitting and Extrapolation of Parameters

Regarded as especially promising (including by Keyfitz, 1984) is the parameterization of the life table l_x by some function, algebraic or transcendental. Once each life table has been fitted to a curve, the sequence formed by each constant is extrapolated, and from these the future age curves are constructed. Numerous analytical forms appear in the literature, starting more than 150 years ago with Gompertz (1825). Four that have been tested are due to Makeham (1860), Perks, unnamed British actuaries, and Pollard-Heligman (Benjamin and Pollard, 1980; Heligman and Pollard, 1980). These have respectively 3, 4, 5 and 8 parameters, with formulae as follows:

Makeham	$u_x = A + BC^x$
Actuaries	$p_x = \dfrac{1}{1 + A - Hx + Bc^x}$
Perks	$u_x = \dfrac{A + bc^x}{kc^{-x} + 1 + Dc^x}$
Pollard-Heligman	$q_x = A^{(x+B)^C} + De^{-E(\ln x - \ln F)^2} + \dfrac{GH^x}{1 + GH^x}$

Other curves are also promising, especially that of Petrioli and Berti (1979). Zaba (1979) and Stoto (1979) modify the Brass method to use four constants rather than two in the regression, so improving the fit at the youngest and oldest ages. This author has not applied these to projection.

Fitting the four curves to the life tables by least squares is not straightforward, and turned out to exceed the author's programming ability. He had to call on help from Sergei Scherbov of IIASA, who produced the fits. Extracted from Scherbov's work is the set of departures of the fitted from the observed for the 13 life tables (Table 8).

TABLE 8. ROOT MEAN SQUARE ERROR OF FIT TO l_x FOR FOUR FUNCTIONSAT 13 DATES

Year	Makeham	Actuaries	Perks	Pollard
1921	0.033450	.02482	.005108	0.000885
1926	0.03542	0.02600	0.005769	0.001261
1931	0.03292	0.02494	0.005045	0.001019
1936	0.02921	0.02213	0.003843	0.001583
1941	0.02330	0.01777	0.002911	0.001080
1946	0.01960	0.01471	0.001724	0.001206
1951	0.01640	0.01265	0.001370	0.002454
1956	0.01355	0.01005	0.001804	0.002412
1961	0.01208	0.00884	0.002323	0.003046
1966	0.00996	0.00767	0.001796	0.002743
1971	0.00780	0.00565	0.001548	0.001941
1976	0.00574	0.00437	0.001649	0.002207
1981	0.00461	0.00307	0.001611	0.001790
Mean	0.01878	0.01405	0.002807	0.001817

After fitting each of the 13 sets, say of l_x, each parameter would be treated as a time series and extrapolated, then the l_x reconstructed from the extrapolated values of the parameters. The goodness of fit to the past is no guarantee that the future l_x will accord with what comes to pass; it is a necessary condition but hardly a sufficient one. Table 8 shows that the Makeham and Actuaries curves are out of the running — of all the tables, their errors are far greater than those of the other two. Perks is an order of magnitude better than either, and the Pollard curve is on average the best of all. As between the two closest, Pollard is considerably better than Perks up to 1946, after which Perks is somewhat better. What do we conclude from the fact that on the average of the 13 life tables Pollard wins, while on each of the last seven taken by themselves Perks is better? Is Perks preferable because it does better on more recent tables?

TABLE 9. THE EIGHT PARAMETERS OF THE POLLARD-HELIGMAN PARAMETERIZATION OF THE MORTALITY CURVE. FITTING IS BY LEAST-SQUARES TO THE CANADIAN LIFE TABLES FROM 1920-22 TO 1980-82

Year	A	B	C	D	E	F	G	H
1921	0.01431	0.1013	0.2202	0.002907	1.1730	29.98	0.000035	1.106
1926	0.01274	0.0388	0.1717	0.002984	1.5150	30.67	0.000046	1.103
1931	0.01664	0.1826	0.2764	0.002955	0.8464	30.83	0.000041	1.105
1936	0.01230	0.1055	0.2171	0.002371	1.0000	33.43	0.000044	1.104
1941	0.01229	0.2492	0.2963	0.001955	0.3672	40.12	0.000041	1.105
1946	0.00801	0.2197	0.2944	0.001309	0.1679	47.67	0.000047	1.102
1951	0.00353	0.0622	0.1987	0.000928	0.2907	48.07	0.000047	1.101
1956	0.00952	0.5614	0.4928	0.000536	0.0186	48.00	0.000047	1.101
1961	0.00264	0.2505	0.3556	0.000420	0.0001	30.00	0.000050	1.099
1966	0.00247	0.3167	0.3963	0.000466	0.0000	29.97	0.000048	1.099
1971	0.00334	0.4702	0.4625	0.000440	0.0303	30.04	0.000052	1.097
1976	0.00141	0.4702	0.5362	0.000447	0.0035	26.11	0.000050	1.097
1981	0.00006	0.5310	1.1650	0.000343	0.1055	8.81	0.000046	1.097

Beyond Fitting: How to Extrapolate the Parameters

Closeness of fit to the separate life tables does not by itself solve the projection problem, as seen in Table 9 and the charts of the time series of the several parameters, shown here for the Pollard fit. For rather few of the curves would a straight line do for the projection, and in some there is no discernable trend.

Though overall the most hopeful of the several parameterizations proposed for mortality is that of Pollard and Heligman, it offers difficulties for projection. Table 9 and the graphs give an indication of the sensitivity of the future parameters to the past period that is chosen to start the extrapolation. Parameter D, for example, if projected from 1921-56 would show a sharp downward tendency; if projected from 1956-81 would be very nearly level. Similar remarks could be made about their parameter F, while on the other hand, G and H show a trend that would vary less with respect to the past interval from which one extrapolates.

Thus the problem of selection of the base period reappears in this quite different projection method from that of Tables 1 through 8. Parameterization cannot avoid such decisions as whether 1986-2021 will be like 1976-81, or like 1951-81, or like 1921-81.

The Brass curve, having only two parameters to be fitted to the time series, gives time series that are more readily extrapolated.

Projection of Mortality Trend by Regression

Can one bypass such fitting and extrapolation, and simply project the trend of the past 60 years in the rate of improvement of life expectancy? The regression of the improvement in life expectancy against time in calendar years is given by

Constant	7.396
Coefficient of X	-0.003
Std Error of Coefficient of X	0.012
Std Error of Y Estimated	0.750
r Squared	0.006

The coefficient of X turns out to be negative, -0.00313, but the amount is so small that one hardly needs calculation to be assured that it is not significant; in fact, it is only about one-quarter of its standard error.

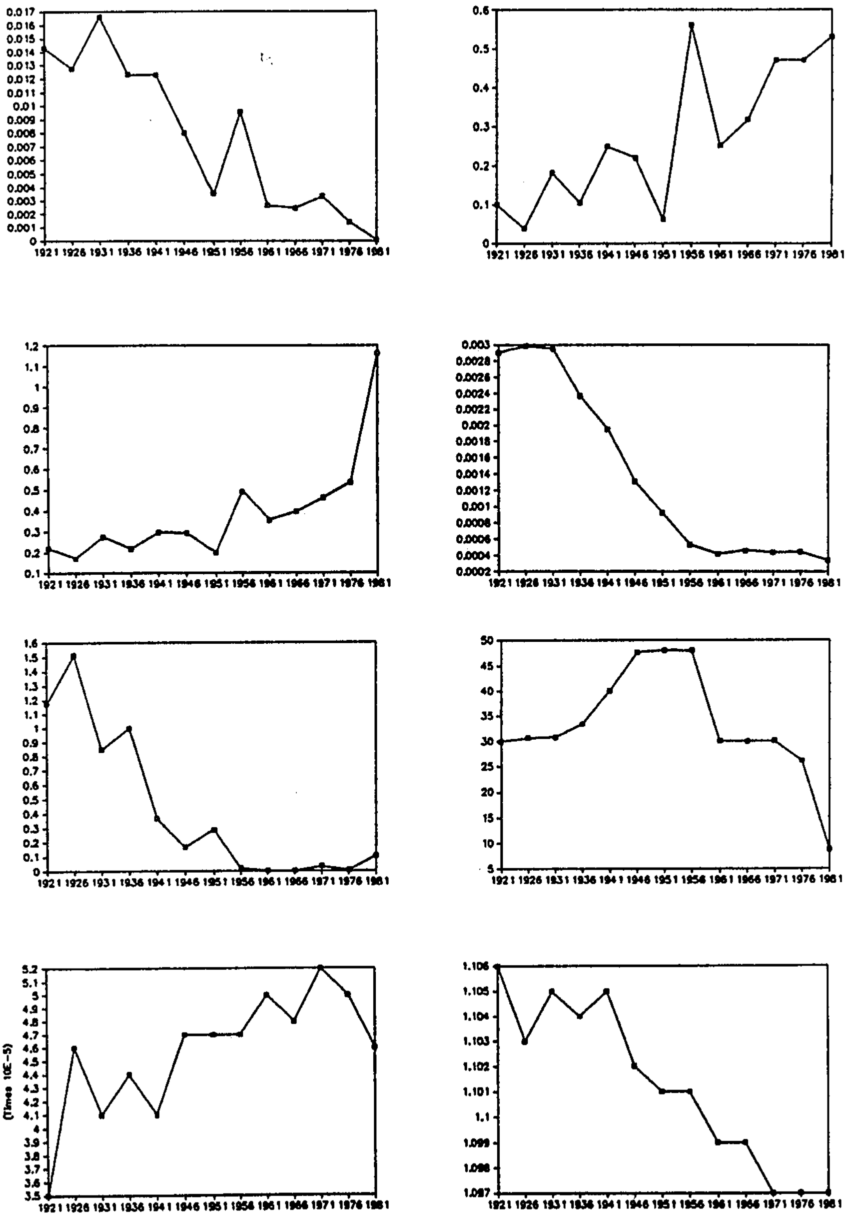

FIGURE 1. TIME SERIES OF THE EIGHT PARAMETERS OF THE POLLARD-HILIGMAN CURVE, SHOWING DIFFICULTY OF EXTRAPOLATION

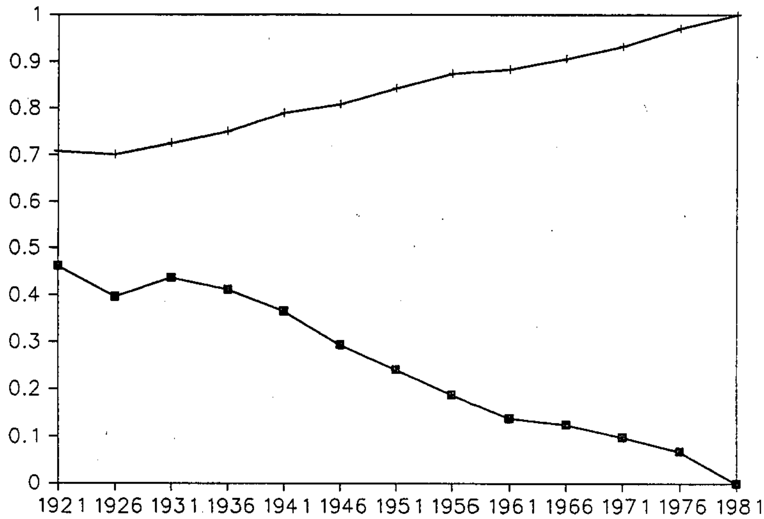

FIGURE 2. TIME SERIES OF THE TWO PARAMETERS OF THE BRASS FITTING TO LOGITS, THE TOP CURVE BEING ß, THE BOTTOM ALPHA

This could be elaborated in various ways. One would be by using GNP as an independent variable. That would be unlikely to secure significance, and even if it did it would add the burden of estimating the GNP for the future. Another elaboration would be to take the trend age by age, but further pursuit of the regression option does not seem worthwhile. In sum, one had best stop looking for a trend in the rate of mortality improvement.

Take Advantage of Serial Correlation?

Another possibility is to make use of serial correlation. A considerably armoury of techniques exists that would allow progression from the last mortality table to the one beyond, and then to one more, and so on. Ronald Lee (1974) has had success along this line, as have McNown and Rogers (1990). Yet with the Canadian data, one is discouraged by finding that the regression of each item on the preceding item in the series of five-year improvements in the $\overset{\circ}{e}_0$ does not help greatly. For the years 1921 to 1981, the resulting correlation and regression are as follows:

Constant	1.039
Coefficient of *X*	0.184
Std Error of Coefficient of *X*	0.331
Std Error of *Y* Estimated	0.776
r Squared	0.033
r	0.180
Mean increase over 12 intervals	1.292

The standard error of the *X*-coefficient is nearly double the coefficient. The coefficient of correlation is the square root of 0.033 or 0.18. The regression shows that one can take the increase in $\overset{\circ}{e}_0$ for 1981-86 to be equal to the increase of 1976-81 times 0.184982 plus 1.039076, but the standard error of this would be 0.776. This is better than the simple trend of improvement, and favours the approaches of Lee and McNown and Rogers.

Recognizing Cause of Death

Whatever the method and period chosen, performing the projection by individual causes has been strongly recommended (Caselli, 1991; Keyfitz, 1984; McNown and Rogers, 1990). In a time when infectious disease was important — and showed a different trend and different age incidence from chronic disease — the case for the recognition of causes was indeed strong. This still applies in many of the less developed countries. Yet when and where infectious diseases have smaller effect on mortality, and the age impact of the chronic diseases does not differ much from one to another, the usefulness of breaking down the calculation by cause is considerably diminished. For the presently less-developed countries, however, the recognition of cause would clearly increase accuracy.

Summary

Lacking significant trend, one can only use the historic average of amount of improvement. Study and comparison have been carried out on the various ways of projecting mortality, and the choices seem to be as follows:

1) Extrapolating each age separately. This can be done on any of the life table functions, l_x,. q_x,, M_x, etc. If on the l_x or q_x transforming by the logit function, extrapolating, and then transforming back will at least meet the minimum requirement that the results come out between zero and unity; so will taking the future q_x or $1-l_x$ in a geometric ratio obtained from some past period.
2) Fitting each past point of time with a suitable function, projecting each of the parameters treated as a time series, then reconstructing the future curves.
3) The Brass method: transforming to logits, regressing on a standard population, then extrapolating the time series of regression coefficients.
4) Regression over time on other series such as GNP per capita.
5) Auto-regression of the series on itself.
6) Performing any of the above by individual causes, then assembling the causes.

Of the above, (4) and (5) show discouragingly low correlations on the Canadian data, and (6) is appropriate in an epoch and for countries where infectious disease is common, since it is distributed over different ages from chronic disease. In Canada, as in other advanced countries, infectious disease has diminished greatly, so most of the important causes of death now are chronic, and these have similar distributions. The usefulness of differentiating causes for projection purposes in countries of low mortality is much diminished by the similarity of those age distributions.

There is no obvious trend in the rate of improvement of mortality — one cannot say that mortality is tending to improve faster as time goes on. Not being able to project a trend, one is reduced to determining an average rate of improvement, and that comes down to deciding from what past period one ought to calculate that average. One obvious choice is the whole record — the 13 life tables that are available for

Canada from 1920-22 to 1980-82. Other choices are the last half of the period, which shows somewhat more improvement than the first half, or the most recent interval in our data, from 1975-77 to 1980-82, which shows phenomenal improvement. To the series can now be added 1985-87. More refined might be weighting the recent life tables more heavily than the earlier ones.

The curve fitting approach has been strongly recommended but little used. Two curves that provide good fits have shown up: that of Pollard and Heligman and that of Perks, the former better for the whole period 1921-81, and the latter better for the latter half of the period. The difficulty comes in projecting the time series of individual parameters, in several of which the future would be crucially dependent on what part of the past one works from. This seems to apply less to the Brass method than to the others; at least its two constants alpha and beta seem to exhibit a steady trend.

A useful compilation of what it is that national offices do to produce official forecasts is provided by Cruijsen and Keilman (1989). They mostly avoid such sophistication as fitting parameters.

Given all this, what is the recommendation? Especially if simplicity in the explanation to the public is a consideration, one could project the q_x by a geometric progression whose initial point is the last existing life table, and whose ratio is the average ratio of change over five year periods in the historic record. If explanation to the public is less important, then the Brass method seems better because it is less affected by the truncation of the table at the highest ages. The methods of Lee (1974) and Petrioli and Berti (1979) remain to be compared with the methods experimented on here.

To repeat the main conclusion, no trend calculation, or regression on economic variables, seems able to forecast the future, that is to capture the extraneous element — technical advance: antibiotics for the 1940s and 1950s, new ways of preventing and treating heart disease in the 1970s — and new fashions in behaviour.

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