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METROPOLITAN POPULATION DECONCENTRATION IN CANADA, 1941-1976

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Résumé — Cette étude donne un compte rendu des changements en densité de population à une distance du centre de ville, tels qu'ils ont été mesurés par la fonction exponentielle négative, pour 20 régions métropolitaines canadiennes durant la période 1941 à 1976. On utilise une estimation à deux points pour déterminer la fonction exponentielle négative à partir de la statistique publiée et des cartes géographiques disponibles. La validation à l'aide des estimations de régression indépendantes de tract de recensement suggère que les estimations à deux points fournissent des données utiles pour décrire la structure spatiale d'ensemble de la répartition résidentielle urbaine.

Deux aspects importants de la dynamique de population urbaine au Canada y sont documentés. En premier lieu, les densités de population de centre de ville ont constamment décliné durant la période 1941 à 1976, sans aucune indication d'interruption dans les déclins. Toutes les capitales ont rencontré ces déclins. En second lieu, les populations ont continué à se développer ou à se déconcentrer durant cette période. La déconcentration devient plus forte avec le temps, et surtout elle caractérise les plus grands centres de population.

L'analyse polyvariée des données pour la période 1941 à 1976 montre l'effet de la dimension de la population, de l'âge urbain et de la région: on trouve des effets importants pour chaque variable. En somme, les résultats indiquent des réductions continuelles dans les densités de centre de ville et une croissance plus rapide en banlieue dans les cités centrales.

Abstract — This paper reports on changes in population density with distance from city centre, as measured by the negative exponential function, for 20 Canadian metropolitan areas during 1941 to 1976. Two-point estimation is used to determine the negative exponential function from available published statistics and maps. Validation with independent census tract regression estimates suggests that the two-point estimates provide useful data for describing the overall spatial structure of urban residential distribution.

Two important aspects of urban population change in Canada are documented. First, central city population densities have consistently declined during 1941 to 1976, with no indication of cessation in the declines. All metropolises have experienced these declines. Second, populations have continued to spread out or deconcentrate during this period. Deconcentration becomes stronger over time and especially characterizes the larger population centres.

Multivariate analysis of the 1941 to 1976 data shows the effect of population size, city age and region: significant effects are found for each variable. Overall, the results indicate continued reductions in central city densities and faster growth in the suburbs than in the central cities.

Key Words — **population distribution, suburbanization, urban ecology, urban sociology, demographic methods**

Introduction

Urbanization and suburbanization have been two of the most distinctive features of population distribution during the past century. Canada has been no exception to these powerful historical forces of population change. In 1825, Montreal and Quebec had about 32,000 and 22,000 inhabitants respectively, while Saint John, New Brunswick, had about 8,100 inhabitants and Halifax had approximately 15,000 persons (Stone, 1967:14). About seven per cent of Canada's 1.1 million residents in 1825 lived in these four urban centres. Historical census data, using somewhat varied definitions of the urban population, show about 20 per cent of the population residing in urban areas by 1871; this percentage increased to 37 per cent in 1901, 54 per cent in 1941, 72 per cent in 1966, and reached 75 per cent by 1976 (Urquhart, 1965: Series A15-19; Canada, 1978:137). The rate of metropolitan growth has thus slowed down in recent years (with apparently even slower rates of growth during 1976-81) (Stone, 1982), evidencing a peaking of urbanization of Canadian society.

Suburbanization is more difficult to chart statistically, even though it seems to be an aspect of population that is clearly apparent. Indeed, a review of the literature shows few descriptions of population suburbanization in Canada. Latham and Yeates (1970) and Yeates and Garner (1971:274) demonstrated that the Toronto metropolitan area deconcentrated or suburbanized from 1951 to 1963 and that the process of deconcentration conformed to patterns in other North American cities. Frankena (1978), in his appraisal of various exponential functions and methods of estimation, also provided evidence that the Toronto metropolitan area was comparatively deconcentrated in 1971; the values he reports show a moderately shallow gradient of population density with distance from Toronto's central business district. An unpublished paper by Hill (no date) provides a useful set of estimates for Canadian cities from 1951 to 1971, along with preliminary analysis about associated factors. The purpose of this paper is to add to the few empirical studies that exist and to analyse the most recent trends in Canadian population suburbanization.

Events during the 1970s provide a variety of reasons for suspecting that population suburbanization might have changed direction or, at minimum, retarded its rates of change during the past decade. First and perhaps foremost, increases in the real cost of operating the private automobile act to increase transportation costs for suburban workers with central city employment, although Canada's oil pricing helped to mitigate the situation somewhat compared to other industrialized countries; that is, increases in transportation costs result in greater minimization of the journey to work and, for those with central city employment, residential location closer to the city core.

Second, income improvements have been implicated in encouraging population suburbanization in several empirical studies (Muth, 1969). In addition, because average real family income rose less during the 1970s, it would be expected that population movement to the suburbs might have slowed down as a result of this factor.

Third, the growth of single-person households — through delayed marriage, a greater proportion never marrying and increases in divorce — represents an additional consideration since primary individuals are less likely than families to compete for traditional suburban housing. Growth of primary individual housing and smaller average household size, coupled with increases in rental housing and condominium redevelopment, has been a special characteristic of Canadian urban areas (Kalbach and McVey, 1971:314, 330; Stone, 1982) and fosters the development of the central core of cities.

Fourth, while there is, for the moment, no recent evidence on employment suburbanization (Muller, 1976, documents trends prior to 1970), we suspect that it slackened during the 1970s as a result of a decreasing rate of growth of metropolitan areas and a decreasing rate of expansion of truck transport (Mills, 1972b:95) and would therefore have retarded suburban population movement. Other works, such as Bourne and Harper's (1974) study of future land use in cities of central Canada, forecast a redevelopment of the urban core, with increased high-density residential land use. Hence, we propose to chart changes in Canadian suburbanization for recent decades, from 1941 to 1976, with the general expectation that population suburbanization is a powerful trend that may have weakened somewhat in the past several years.

Data and Methods

Population suburbanization is measured here by the negative exponential function—a mathematical expression that has found widespread empirical and theoretical support since it was first suggested by Colin Clark in 1951. It was Clark's thesis that population density in a metropolis declines as a negative exponential function of distance from city centre, or in mathematical notation:

$$d(x) = de^{-ax} \tag{1}$$

where $d(x)$ represents population density at distance x from city centre, d is the intercept, a is the density gradient of the function and e is the Naperian constant. For an urban area of limited boundaries, the density gradient a may be interpreted as the reciprocal of the modal distance of the population from the centre (Edmonston and Davies, 1976). When population density is measured by the negative exponential function, then d may be interpreted as the hypothetical population density at the exact centre, and a indicates the slope or the gradient of population density. Diminutions in the density gradient, then, mean that an urban population is becoming more dispersed or spread out over the urban land area. Thus, for example, a = 1.0 indicates that a highly concentrated urban population with a sharp drop in population density from city centre (characteristic of the typical hoof-and-foot city of the nineteenth-century), while a = 0.0 represents a dispersed urban population with equal population density over the land area (the ideal type of multi-nucleated, freeway-dominated urban areas).

The empirical usefulness of the negative exponential function has been widely demonstrated since its introduction by Clark in 1951. Typically, empirical studies have collected population and land area data by census tracts or other small areal units, calculated the distance of each census tract from the city centre, and then regressed the natural logarithm of population density on distance from city centre in order to estimate the a and b coefficients of the negative exponential function. Such procedures have uniformly yielded a close fit of the negative exponential function.

A disadvantage of working with census tract data for estimating the negative exponential function is that estimating land areas and distance is quite time consuming. Moreover, if the negative exponential function is an accurate representation of population density with distance from city centre, then two-point estimation of the negative exponential function is applicable.

Two-Point Estimation

Estimation of the negative exponential function for population density by distance from city centre provides values for the density gradient and the central density. This estimation can be made using available data on two areas: the central city and the total metropolitan area. The required data consist of population, land area, degrees of land area excluded from the urban settlement, and an adjustment for the geometric shape of the urban area.

Although using the negative exponential function in this paper, we believe that it is important to note a limitation about its analytical value in general. Dacey (1971) showed that measures for several population distributions, including the negative exponential, are difficult to evaluate spatially and, even if evaluated, result in expressions that are complex to the point of inhibiting further analysis. Therefore, some care should be taken in differentiating between the goals of models which use the negative exponential function. On the one hand, if the purpose of a study of urban population is to identify a simple function which provides a reasonably close approximation of observed population density, then the objection noted above is not critical. However, if a study is motivated by the need to provide a density function which will serve as one component of a model describing spatial distributions of other urban phenomena, then the limitations noted above are critical. The goals of this paper correspond with the former objective; hence the pitfalls noted by Dacey (1971) are avoided.

Suppose that the negative exponential function is an accurate representation of population density within a metropolitan area. Then, if we use equation (1) the number of people within the circular area of radius z would be:

$$N(z) = 2\pi \int_0^w xde^{-ax}dx \quad (2)$$

where $N(z)$ is the number of people within the circular area from the city centre out to z distance from city centre, 2π is the circumference of the area, d and a are the parameters to be estimated, and the integration is taken over x. For a partial circle in which a portion of the land area is excluded from potential residential development, 2π may be replaced by $2\pi - \varnothing$, where $\varnothing$ represents the number of radians excluded from the circular area.

Integration of equation (2) yields:

$$N(r) = \frac{2d\pi}{a^2}\{1-(1+ax)e^{-ax}\} \quad (3)$$

As is, equation (3) is not solvable since it contains two unknowns. Consider, then, two concentric circles with radii r_1 and r_2 where $r_1 < r_2$. Two equations may be written to express the number of people residing within r_1, the central city, and r_2, the total metropolitan area from city centre. This provides a two equation system, expressed as $N(r_1)$ and $N(r_2)$, that can be solved for a and b.

First of all, d can be derived from equation (3) written in terms of r_2:

$$\mathrm{d} = \frac{N(r_2)a^2}{2\pi\{1-(1+ar_2)e^{-ar_2}\}} \quad (4)$$

Substituting for d in a form of equation (3) that involves r_1:

$$N(r_1) = \frac{N(r_2)\{1-(1+ar_2)e^{-ar_2}\}}{\{1-(1+ar_2)e^{-ar_2}\}} \quad (5)$$

Subtracting $N(r_1)$ from each side of equation (5), the equation may be written as a nonlinear function of a. Various algorithms are available for solution of nonlinear equations; Mueller's iterative procedure of successive bisection and inverse parabolic interpolation is used here for solution a to five significant digits. Knowing a, d is calculated by the substitution of a in equation (3).

The average distance to the central city or metropolitan border (r_1 or r_2, respectively) can be calculated by the simple approximation of the coded geometric shape of each area (Edmonston, 1975:39-41, presents a detailed calculation of this method). Then, from formulae for elementary shapes (circle, hexagon, square or several types of rectangles), the average distance can be calculated directly from the known land area. Thus, for instance, the average distance from city centre to the boundary, r, of a square is $(.5611) \cdot (\sqrt{A})$, where A is the land area. Experimentation with various shapes shows that this approximation is relatively robust except for rectangular areas where the ratio of the length to width exceeds about six. For purposes here, each area was coded by its geometric shape, and then the appropriate conversion formula was used to derive the average distance to the boundary from the land area.

Six pieces of data are required for the estimation of the negative exponential function for a metropolitan area at one time point: two population sizes, two land areas, degrees excluded from the settled area and geometric shape. Population data are available from published sources for the central city and total census metropolitan area (CMA) for all time periods — 1941, 1951, 1956, 1961, 1966, 1971 and 1976 (Canada, 1953, 1962, 1967, 1973, 1977). In order to obtain the most reliable estimates, metropolitan areas with a suburban population less than 10,000 residents were excluded. This resulted in the exclusion of Regina, Saskatoon and Thunder Bay from the observed CMAs. Land areas are available from the same published sources for comparable areas from 1956 to the present. For 1941 and 1951, land areas of central cities and total CMAs were calculated with a polar planimeter scaled to the Dominion Bureau of Statistics census maps. In a few cases, the map scale was missing; comparable 1961 maps were then consulted in order to calculate the 1941 or 1951 map scale. The degrees excluded from the city land area and the urban shape were coded directly from census maps for each time period.

Following the procedures outlined and with the data described, two-point estimates of the negative exponential function were made for 20 Canadian metropolitan areas, 1941 to 1976. Table 1 presents the estimates of the central densities, and Table 2 shows the estimates of the density gradients.

Validation of Two-Point Urban Shape Method

The method presented offers a useful, compact and efficient estimation of the negative exponential function for urban population data. This section gives a brief validation of the method by estimating the negative exponential function with traditional regression analysis of census tract data. (For an alternative approach, White [1977] offers justification of the two-point method through a simulation study.)

Several prior sources offer regression estimates for the negative exponential function for Toronto in 1951 and 1961 (Latham and Yeates, 1970) and Toronto in 1971 (Frankena, 1978). In addition, I have applied the regression approach to 1976 census tract data for Halifax, London, Thunder Bay and Winnipeg. Table 3 shows the comparisons of the two-point and the census tract regression estimates. Note, first of all, that the negative exponential offers a close fit to population density with distance from city centre, as evidenced by the moderately high multiple correlation coefficient (R). Winnipeg shows the lowest value of R, and, even in this case, distance explains over 50 per cent of the variation in population density. For all cities in 1976, a quadratic exponential function was estimated, and only London displays a significant departure from a simple negative exponential function with a statistical significant quadratic term. Estimates reported by Frankena (1978) for Toronto in 1971 show the fit of a third-order

TABLE 1. CENTRAL DENSITIES FOR METROPOLITAN AREAS, CANADA, 1941-1976

Metropolitan Area	Year 1941	1951	1956	1961	1966	1971	1976
Calgary, Alberta	----[a]	39,000	----	----	----	----	----
Chicoutimi-Jouquiere, Quebec	----	----	----	----	----	2,200	2,400
Edmonton, Alberta	15,000	18,000	----	45,000	45,000	29,000	24,000
Halifax, Nova Scotia	53,000	54,000	42,000	37,000	31,000	23,000	20,000
Hamilton, Ontario	57,000	55,000	25,000	24,000	24,000	23,000	22,000
Kitchener, Ontario	----	----	8,200	10,000	13,000	5,500	6,100
London, Ontario	38,000	36,000	26,000	30,000	35,000	29,000	32,000
Montreal, Quebec	320,000	89,000	71,000	65,000	57,000	51,000	41,000
Oshawa, Ontario	----	----	----	----	----	8,800	10,000
Ottawa-Hull, Ontario-Quebec	11,000	10,000	19,000	22,000	22,000	20,000	18,000
Quebec, Quebec	76,000	71,000	67,000	59,000	50,000	15,000	13,000
Saint John, New Brunswick	26,000	21,000	20,000	20,000	16,000	7,300	5,500
St. Catharine's-Niagara, Ontario	----	----	----	----	----	6,700	7,900
St. John's, Newfoundland	----	160,000	72,000	75,000	75,000	56,000	48,000
Sudbury, Ontario	----	----	6,900	7,300	7,700	3,600	3,400
Toronto, Ontario	94,000	69,000	53,000	46,000	41,000	39,000	32,000
Vancouver, British Columbia	29,000	29,000	26,000	24,000	24,000	23,000	20,000
Victoria, British Columbia	12,000	12,000	11,000	9,900	9,800	11,000	11,000
Windsor, Ontario	64,000	53,000	34,000	33,000	36,000	27,000	24,000
Winnipeg, Manitoba	46,000	41,000	29,000	27,000	23,000	41,000	41,000

[a] Data unavailable or suburban population less than 10,000 residents. This results in no estimates for Regina, Saskatoon, and Thunder Bay.

TABLE 2. DENSITY GRADIENTS FOR METROPOLITAN AREAS, CANADA, 1941-1976

	Year						
Metropolitan Area	1941	1951	1956	1961	1966	1971	1976
Calgary, Alberta	-.--[a]	1.32	-.--	-.--	-.--	-.--	-.--
Chicoutimi-Jonquiere, Quebec	-.--	-.--	-.--	-.--	-.--	.31	.32
Edmonton, Alberta	.99	.81	-.--	.91	.83	.60	.53
Halifax, Nova Scotia	1.67	1.45	1.16	1.02	.89	.53	.49
Hamilton, Ontario	1.16	.99	.59	.54	.51	.46	.44
Kitchener, Ontario	-.--	-.--	.62	.63	.64	.35	.35
London, Ontario	1.62	1.35	1.02	1.01	1.02	.83	.84
Montreal, Quebec	1.33	.64	.50	.43	.38	.34	.30
Oshawa, Ontario	-.--	-.--	-.--	-.--	-.--	.45	.46
Ottawa-Hull, Ontario, Quebec	.45	.33	.58	.58	.52	.45	.41
Quebec, Quebec	1.40	1.22	1.11	.97	.83	.41	.37
Saint John, New Brunswick	1.08	.92	.86	.81	.71	.46	.39
St. Catherine's,-Niagara, Ontario	-.--	-.--	-.--	-.--	-.--	.18	.20
St. John's, Newfoundland	-.--	1.90	1.19	1.14	1.08	.82	.72
Sudbury, Ontario	-.--	-.--	.65	.63	.63	.38	.37
Toronto, Ontario	.57	.43	.33	.28	.24	.22	.19
Vancover, British Columbia	.49	.41	.35	.30	.29	.26	.23
Victoria, British Columbia	.49	.39	.34	.29	.26	.29	.26
Windsor, Ontario	1.04	.82	.61	.60	.59	.47	.45
Winnipeg, Manitoba	.98	.84	.66	.58	.52	.67	.66

[a]Data unavailable or suburban population less than 10,000 residents. This results in no estimates for Regina, Saskatoon, and Thunder Bay.

exponential function. Figure 1 shows the third-order function as line (1), the graphic fit to the line (1) is labelled line (2), and the two-point estimate is line (3) for Toronto data in 1971. Even in the case of 1971 Toronto, it appears that the two-point estimate provides an appropriate description of population density patterns for purposes of this research.

Overall, given the vagaries of the spatial definitions, the results of Table 3 show consistency in the two estimation procedures. Several of the metropolitan areas — Halifax, 1971 Toronto, Thunder Bay and Winnipeg — display close agreement between the two estimates. London, Ontario, differs between the two methods, and the actual population density pattern is closer to a quadratic exponential function. Toronto in 1951 also varies for the two estimates, but the actual spatial areas in the two estimates are somewhat different. In short, it appears that when there is an actual underlying negative exponential function to urban population density (and there usually is) and comparable land areas are used, then the two-point estimation provides reasonably similar results to the census tract regression estimation.

A comprehensive set of estimates of the negative exponential function for Canadian metropolitan areas has also been prepared by Hill (no date), who has made estimates for 12 metropolitan areas in 1951, 1956, 1961, 1966 and 1971. His estimates involve population data on the basis of the 1971 census tract boundaries, measured distances to the population centre of each tract, and calculated land areas. Then, the urbanized area was determined for each year by using a population density criterion of 1,000 or more population per square mile, and the negative exponential function was estimated by unweighted ordinary least squares.

I have compared in detail the estimates presented here and those made by Hill. First of all, there is a clear constant difference in the two series: the density gradients estimated by Hill are usually more shallow than the ones reported here. It appears that this exists because Hill's urbanized area is more limited spatially — it does not include portions of the rural-urban fringe — and unweighted regression data differentially favour distance zones with the most numerous census tracts (Frankena, 1978). Second, there is moderate random variation between the two sets of estimates. For 12 metropolitan areas in 1971, for example, there is a correlation co-efficient of .49 between the two sets of density gradients. Although this implies the need for caution in placing great reliance on a simple estimate of the density gradient for a particular city, it is worth emphasizing that both Hill's estimates and those shown here are quite similar when the spatial data are consistent. Both Saint John and Victoria have similar urban borders in Hill's data and in these estimates, and the density gradients are quite close to each other for all time periods.

As a final note on the Hill data, the temporal change is similar in both sets of estimates. First, both data sets show a constant decrease in the density gradient and central density. Thus, there is no controversy in the conclusion that Canadian metropolitan areas are becoming less concentrated and more spread out over time. Second, both data sets display a decreasing rate of decline in the density gradient over time. Hill's data (calculated from his Table 1) show an average annual density gradient decline of .010 for 1951-61 and .009 for 1961-71. For the same 12 cities, my data (based on Table 2) indicate declines of .021 for 1951-61 and .019 for 1961-71. The absolute amount of decline differs because the actual density gradients for the two series vary; what is important here is that both show declines in the density gradients and a diminuation of the rate of decline over time. So while acknowledging that both the type of estimation (regression analysis and two-point procedures) and the nature of the spatial data affect the negative exponential function estimates, it must be emphasized that the substantive interpretations do not differ greatly.

TABLE 3. COMPARISON OF ESTIMATED NEGATIVE EXPONENTIAL FUNCTIONS FROM TWO-POINT METHOD AND CENSUS TRACT REGRESSION FOR SELECTED CANADIAN URBAN AREAS

Metropolitan Area	Negative Exponential Function Coefficients[1]		Census Tract Regression	
	d	a	Number of Tracts	Multiple R
Halifax, Nova Scotia, 1976				
Two-Point Method	20,000	.49	---	.--
Census Tract Regression	20,000	.46	13	.88
London, Ontario, 1976				
Two-Point Method	32,000	.84	---	.--
Census Tract Regression[2]	28,000	.64	56	.87
Toronto, Ontario, 1951				
Two-Point Method	69,000	.43	---	.--
Census Tract Regression[3]	110,000	.21	300	.80
Toronto, Ontario, 1961				
Two-Point Method	46,000	.28	---	.--
Census Tract Regression[3]	120,000	.16	300	.80
Toronto, Ontario, 1971				
Two-Point Method	39,000	.22	---	.--
Census Tract Regression[4]	71,000	.28	325	.--
Thunder Bay, Ontario, 1976				
Two-Point Method	13,000	.58	---	.--
Census Tract Regression	12,000	.67	12	.90
Winnipeg, Manitoba, 1976				
Two-Point Method	41,000	.66	---	.--
Census Tract Regression	44,000	.71	119	.73

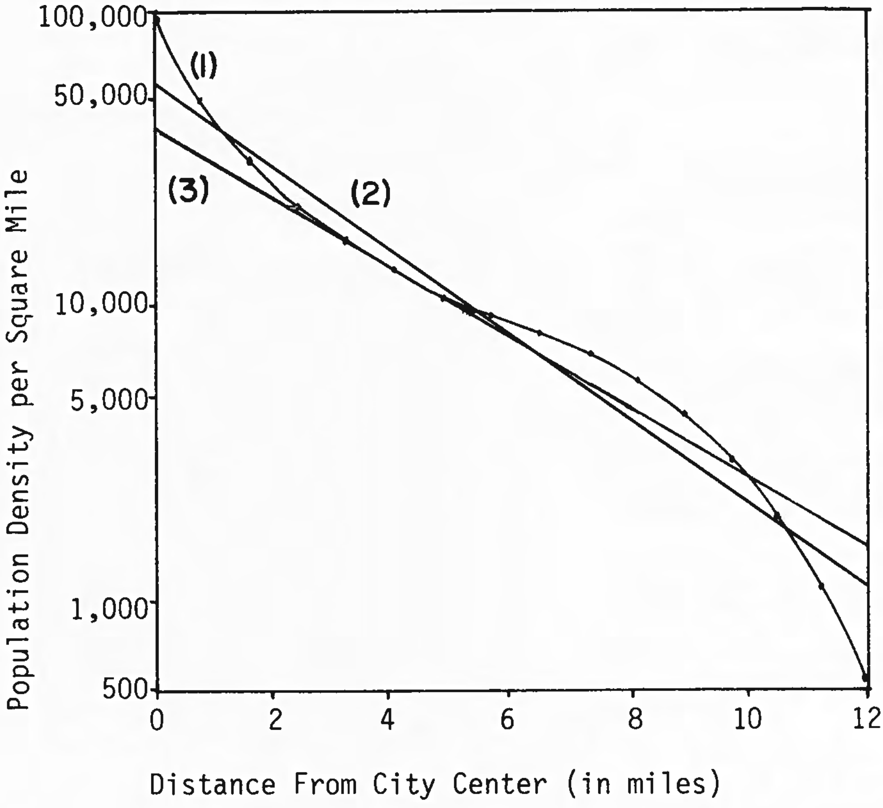
[1] negative exponential function, $\ln d(x) = \ln d - ax$, where d(x) population density, x is distance from city center, d is central density, and a is the gradient

[2] a quadratic exponential, $\ln d(x) = \ln d - ax + bx^2$, displays statistical significance at the .05 level for all three parameters: d = 19,000, a = .65, and b = .013

[3] Latham and Yeates, 1970: 274

[4] Frankena, 1978: Table 1. This is a graphic fit to the unweighted third-order exponential equation, and hence there is no multiple R available

FIGURE 1. POPULATION DENSITY WITH DISTANCE FROM CITY CENTRE, TORONTO CMA, 1971



Equation (1): unweighted third-order exponential function, Frankena, 1978: Table 1

Equation (2): graphic fit of simple exponential function to equation (1)

Equation (3): two-point estimate of negative exponential function, data shown in Tables 2 and 3

Results

Returning to Tables 1 and 2, several general patterns are noticeable. First, with few exceptions, it appears that metropolitan areas have experienced declines in both central densities and density gradients. Further analysis will document these trends more precisely. Secondly, there are interesting differences, between areas, that invite further investigation—some are more dense and compact than others.

Tables 1 and 2 include 12 areas for which there are estimates for each time period, 1941 to 1976. Selecting only these 12 areas, Table 4 shows the average annual rate of change of central density. For this group of 12 areas, central density has consistently declined, from an average of 69,000 people in 1941 to 23,000 people per square mile in the hypothetical city centre. We also see that, with the exception of 1966-71, the average annual rate of change has somewhat lessened over time. That is, central density continues to decrease but at a somewhat slackened pace in more recent years.

Table 4 shows the average gradients for the same 12 metropolitan areas. Again, there is a pattern of monotonic decreases in the average density gradients. The density gradient of 1.02 in 1941, reflecting more compact cities, decreased by more than one-half to .42 in 1976, and these Canadian metropolises spread out or suburbanized in the 35-year period. The density gradient has also decreased at a slower pace—with the exception of 1966-71—with less than a .01 annual decline in the most recent five-year period. This process generally confirms a pattern of change suggested by Newling (1966), in which he argued that the density would decline at a negative exponential rate. A fit of a negative exponential function to the seven density gradients shown in Table 4 shows an excellent fit ($r = .99$) to the time trend. Application of this model indicates the density gradient declining at a decreasing rate, with a density gradient of .28 forecast for the year 2001. Other factors are important, however, in determining the density gradient, and later analysis offers a fuller multivariate analysis of the time trends.

Figure 2 shows the negative exponential function for the average of these 12 metropolitan areas. It demonstrates the declines in the central density and the decreases in the density gradient at the same time. Overall, there has been a rotation of the negative exponential function at about two miles from city centre, with decreased population density within two miles and broad increases in population density in zones that are two miles and more from city centre. There have been especially dramatic gains in population density in the areas five miles and more from city centre: these areas were generally low density rural population prior to 1951, but by 1976, they showed population densities of about 500 to 3000 people per square mile.

Bivariate Results

Taking all metropolitan areas together, Table 5 presents changes in the central density classified by CMA population size, city age and region. City age is defined here as the number of years since the city reached a population of 50,000 or more inhabitants. Montreal is the oldest city in Canada by this criterion, since it reached a population of 50,000 in 1861. One sees noteworthy variations in the population density at the hypothetical city centre. There have been substantial declines in central density, as evidenced by the overall mean, as well as consistent variations for the three independent variables.

CMA population size shows strikingly higher central densities in the larger areas (500,000 or more population) in 1941. There are relatively similar levels of central density in the two population size groups for less than 500,000 residents, and there has been a decrease in the CMA population size differentials over time. Comparatively small differences exist in central densities by 1976 by CMA population size.

TABLE 4. MEAN CENTRAL DENSITY AND DENSITY GRADIENT FOR 12 METROPOLITAN AREAS, CANADA, 1941-1976[a]

	Central Density		Density Gradient	
Year	Mean	Annual Rate of Change in Preceding Period	Mean	Annual Rate of Change in Preceding Period
1941	69,000	-----	1.02	.---
1951	45,000	-2,400	.82	-.021
1956	35,000	-2,000	.68	-.028
1961	33,000	-380	.62	-.012
1966	31,000	-430	.56	-.010
1971	26,000	-1,000	.45	-.023
1976	23,000	-570	.42	-.005

[a] based on twelve metropolitan areas shown in Tables 1 and 2, for which complete data exist for the 1941 to 1976 period.

City age demonstrates that the oldest Canadian cities had the highest central densities in 1941, although there are no marked differences for cities less than 50 years old in 1941. All city age groups experienced decreases in central density between 1941 and 1976. By 1976, the two youngest age groups have noticeably lower central densities than cities that are 31 or more years of age.

Region displays a varied and interesting set of variations. Three regions — the Maritimes, Quebec and Ontario — share in the temporal decline of central density, with Quebec and Ontario metropolitan areas witnessing particularly sharp drops. The Maritime areas have had lower central densities than the national average and had moderate declines in central densities. The West, including the Prairie provinces and British Columbia, displays relatively low central densities in the earlier years and little change over time.

Suburbanization has continued for 1941-76 in all the categories examined here, as is shown by the changes in the density gradients in Table 6. For all metropolitan areas combined, the overall mean density gradient decreased steadily from a value of 1.03 in 1941 to .42 in 1976. The temporal pattern is consistent with that examined in Table 4 for the cohort of 12 metropolitan areas.

Metropolitan areas with larger CMA population size tend to have lower density gradients,

TABLE 5. AVERAGE CENTRAL DENSITIES BY POPULATION SIZE, CITY AGE, AND REGION FOR CANADIAN CMAs, 1941-1976

Variable	1941	1951	1956	1961	1966	1971	1976
CMA Population Size							
-250,000	42,000	48,000	28,000	28,000	28,000	15,000	15,000
250,000-500,000	37,000	44,000	35,000	35,000	35,000	22,000	17,000
500,000+	208,000	63,000	50,000	45,000	36,000	30,000	26,000
City Age (in years)							
-10	34,000	31,000	30,000	26,000	10,000	6,400	7,500
11-30	36,000	40,000	27,000	27,000	34,000	18,000	14,000
31-50	34,000	34,000	32,000	30,000	34,000	27,000	25,000
51+	164,000	76,000	47,000	43,000	35,000	30,000	26,000
Region							
Maritimes	39,000	77,000	45,000	44,000	41,000	29,000	24,000
Quebec	199,000	80,000	69,000	62,000	54,000	23,000	19,000
Ontario	53,000	40,000	24,000	27,000	28,000	18,000	17,000
West	29,000	30,000	22,000	20,000	19,000	26,000	24,000
Overall Mean	69,000	50,000	35,000	33,000	32,000	22,000	20,000
Sample Size	12	15	14	16	16	19	19

that is, a more dispersed population. Moreover, although all population size groups show suburbanization, we see an especially strong pattern of deconcentration for areas of 500,000 or greater inhabitants, with declines in the density gradient to 1961 levels of about one-third the 1941 levels. It is also striking that larger population centres do not show continued declines after 1961: it appears that the years following World War II witnessed a major change in urban residential spatial structure, but that the rate of change has attenuated somewhat more recently. Smaller CMA population size groups had declines in the density gradient of a lesser amount, with 1976 levels of approximately one-half the 1941 levels. Unlike the largest areas, however, the density gradient has continued to decline in the two smaller population size categories.

City age displays decreases in the density gradient for all age groups. Moreover, there appears to be a consistent relationship, with older cities being more compact in recent years. Multivariate analysis, taking into account the association between city age and CMA population size, will be used to evaluate the separate effect of city age.

All four regions show suburbanization of their metropolitan areas. The Maritime metropolitan areas retain somewhat more concentrated metropolitan areas, while Quebec's areas demonstrate particularly sharp drops in the density gradient over time.

FIGURE 2. POPULATION DENSITY BY DISTANCE FROM CITY CENTRE
FOR 13 CANADIAN METROPOLITAN AREAS FOR 1941-1975 (DERIVED FROM TABLE 4)

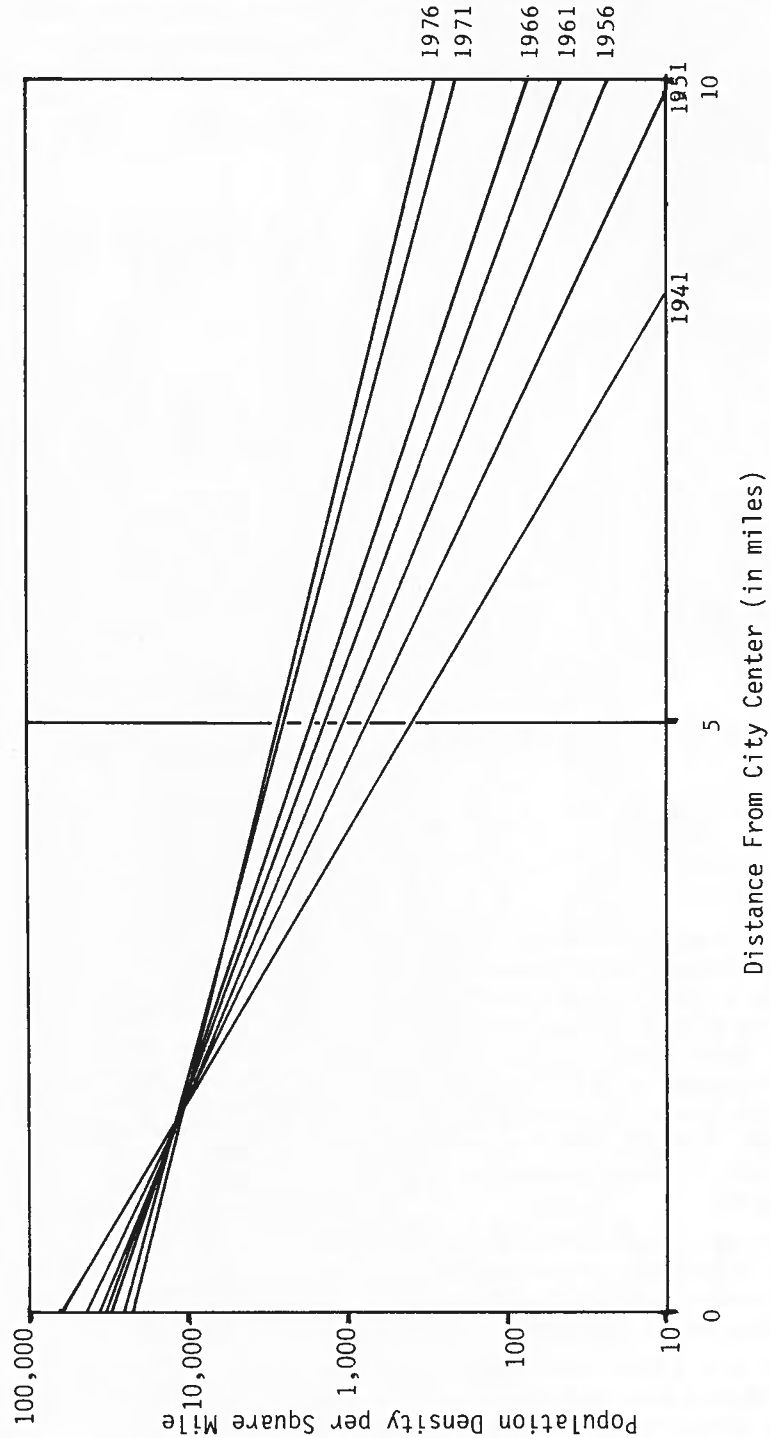


TABLE 6. AVERAGE DENSITY GRADIENTS BY POPULATION SIZE, AGE OF CENTRAL CITY, AND REGION FOR CANADIAN CMAs, 1941-1976

Variable	1941	1951	1956	1961	1966	1971	1976
CMA Population Size							
-250,000	1.11	1.12	.78	.76	.73	.44	.42
250,000-499,999	.73	.85	.74	.71	.67	.54	.47
500,000+	.95	.50	.39	.34	.36	.40	.39
City Age (in years)							
-10	.87	1.07	.70	.67	.63	.29	.32
15-30	1.19	1.15	.74	.70	.68	.48	.39
35-50	.80	.65	.72	.77	.83	.61	.42
55+	1.10	.76	.62	.56	.47	.40	.45
Region							
Maritimes	1.37	1.42	1.07	.99	.89	.60	.53
Quebec	1.36	.93	.80	.70	.60.	.35	.33
Ontario	.97	.79	.57	.64	.62	.42	.41
West	.65	.74	.45	.39	.36	.46	.42
Overall Mean	1.03	.93	.69	.67	.63	.45	.42
Sample Size	12	15	14	16	16	19	19

Multivariate Analysis

Three variables have been shown to be associated with the central density and density gradient of Canadian metropolitan areas: CMA population size, city age and region. In addition, it is apparent that central density and the density gradient vary by time period. This suggests a multivariate analysis in which the relative effect of each of the four factors is estimated, controlling for the influence of the other factors. With multivariate analysis, we would expect to find the following results. Larger population areas are hypothesized to be more dispersed and to have higher central densities. These two conditions arise as a result of a more intense demand for highly accessible space, which increases central densities, while at the same time multi-nucleation of the metropolitan areas encourages a spreading out of the residential population. Other conditions being equal, we expect that older areas have a more compact physical structure, stemming from an earlier period of more compacted residential development and a transportation system designed to serve that spatial distribution. Population size and city age are intertwined in the regional effect, since the Maritimes tend to have older cities while the West has younger cities and both Quebec and Ontario have somewhat larger metropolitan areas. Prior work on regional effects suggests that where there is a lower agricultural demand for land, there are lower land rentals and then more spread out, less dense urban areas (Muth, 1969:174). Further empirical study would be required, however, in order to develop prior expectations about regional influences for Canadian urban areas. This study will therefore be an exploratory analysis of regional variations, rather than an analysis to confirm a particular hypothesis about regional effects. Finally, we hypothesize that the temporal effect is one showing progressively less dense, more suburbanized metropolitan areas in the 1941 to 1976 period.

A particularly attractive model for the multivariate investigation is multiple classification analysis (MCA), since the dependent variables — central density and the density gradient — are continuous and the independent variables can be treated as categorical (Little, 1982). The MCA model is additive and assumes no interaction in this research (it is possible to include interaction in the MCA model with examination of fitted residuals or direct specification of an interactive term). With an additive model, the predicted cell mean is the sum of the overall mean plus the specific categorical effects. This additive nature of the model yields a useful interpretation: each categorical net effect indicates the degree to which that category, controlling for all other independent variables, deviates from the grand mean.

For the estimation of the MCA model, each of the four independent variables is specified as a series of categorical effects. CMA population is considered as three categories: less than 250,000 residents, 250,000 to 500,000 residents and 500,000 or more residents. City age is defined in terms of four groups: less than or equal to 10 years, 11 to 30 years, 31 to 50 years and 51 or more years since reaching a city population of 50,000. A standard regional definition is adopted here: Newfoundland, Nova Scotia, Prince Edward Island and New Brunswick are the Maritimes; Quebec and Ontario are treated as two separate regions; and Manitoba, Saskatchewan, Alberta and British Columbia are the West (there are no metropolitan areas in the Yukon or Northwest Territories). Each census year, 1941 to 1976, is included in the MCA model as a separate categorical variable.

Table 7 shows the MCA estimation of central density for Canadian metropolitan areas. The left-hand column shows the four categorical variables. In the second column are the number of observations for each category, with a total sample size of 119. The third column displays the categorical mean which, when subtracted from the grand mean of 34,600, yields the unadjusted deviation of the particular category. In the final column are the adjusted deviations of the category that result from the MCA estimation.

First of all, one notes that the multiple R is comparatively high, indicating that the variables in the MCA model account for about one-third of the variation in central density. The actual F-statistics are not shown since all variables in the model are statistically significant at the .05 level.

All the independent variables demonstrate the expected pattern. Larger population areas have greater central densities. The adjusted deviations suggest a "life cycle" interpretation in which areas intially increase their central densities as they grow from small to moderate-sized urban areas, and then experience stronger gains in central densities as they become major metropolises.

City age shows a consistent pattern of higher central densities in older cities. Moreover, the impact of city age appears to be unaffected by other variables since the adjusted deviations display a strong, monotonic relationship. The overall variation by city age is considerable: the oldest cities tend to have a central density of about 10,000 people per square mile more than the youngest cities.

Region was included in the MCA equation with no hypothesized association. Table 7 makes

TABLE 7. MULTIPLE CLASSIFICATION ANALYSIS OF CENTRAL DENSITIES, 1941-1976

Variable	Number	Category Mean	Deviations Unadjusted	Deviations Adjusted
Metropolitan Population Size				
-250,000	54	28,900	-5,700	-7,500
250,000-500,000	27	31,300	-3,300	-2,300
500,000+	30	47,900	13,300	16,000
City Age (in years)				
-10	20	25,800	-8,800	-6,800
11-30	27	27,100	-7,500	-1,400
31-50	24	30,400	-4,200	910
51+	40	46,600	12,000	3,800
Region				
Maritimes	20	42,300	7,700	15,900
Quebec	16	65,000	30,400	21,200
Ontario	51	27,000	-7,600	-5,700
West	24	24,000	-10,600	-15,000
Year				
1941	12	68,400	33,800	37,600
1951	15	49,500	14,900	17,600
1956	14	33,900	-660	290
1961	16	32,800	-1,800	240
1966	16	31,300	-3,300	-3,600
1971	19	21,500	-13,100	-15,300
1976	19	19,400	-15,200	-19,700
Sample Size	111			
Grand Mean		34,600		
Multiple R				.639

evident that there are significant regional variations in central densities that persist after controlling for other variables. In particular, the adjusted deviations show higher central densities in the Maritimes and Quebec compared to somewhat lower central densities in Ontario and the West. Several possible explanations might be advanced based on theoretical models of urban spatial structure (Mills, 1972b: chapter 4), but data are not at hand for making any empirical explanation of the regional pattern. Further research would be required to test some of the more plausible explanations, such as that (1) there are higher land rents in Ontario and the West, (2) average family income is greater in metropolitan areas of Ontario and the West or

(3) more developed, cheaper urban transportation systems exist in Ontario and the West. Future research would benefit from a literature developing on Canadian and American cities (Mercer, 1979; Goldberg and Mercer, 1980) which suggests that Canadian cities have a more compact urban form, taking population size into account. Conclusions about regional differences require a well-specified model of urban residential distribution, collection of additional variables, and appropriate statistical estimation. We note, at this point, that striking regional variations exist, and they persist over time while controlling for population size and city age.

The effects of time period show, in Table 7, the pervasive temporal decline in central densities. Decreases in central density have not occurred, therefore, because Canadian metropolises have become bigger and older. There is instead a massive historical trend toward less dense central city populations. The change has been dramatic: controlling for other variables, the central density is 72,200 (the grand mean plus the adjusted deviation: 34,600 + 37,600) in 1941 and 15,000 in 1976, or a 1976 level of less than one-fourth the 1941 level. Moreover, it appears that the decrease in central density continues unabated over time. There is certainly no indication of a levelling of central density, nor that there will be further decreases from the 1976 level.

Turning to Table 8, there are comparable results for population deconcentration or suburbanization, as indicated by the density gradient. All independent variables are statistically significant at the .05 level for the F-test, and about 57 per cent of the variation in the density gradient is explained by the MCA equation.

CMA population size has a consistent effect on the density gradient in the expected direction. Larger population is associated with more dispersed populations. Examining the relative size of the adjusted deviation from the grand mean, smaller urban areas of less than 250,000 have a compact distribution ($b = .66 + .12 = .74$) while larger centres are considerably more deconcentrated ($b = .66 - .23 = .43$).

City age shows the expected association with the density gradient after controlling for other factors. Note the difference between the unadjusted deviations which exist because city age is associated with other variables in the model, especially population size. For example, older cities tend to have larger population size, and only with multivariate analysis can the independent influence of city age emerge. The resulting pattern in the adjusted deviations is clear: older cities tend to have more compact spatial structures.

Region has a statistically significant effect on the density gradient, showing especially compact centres in the Maritimes, modestly more compact areas in Quebec, modestly more dispersed centres in Ontario and more dispersed areas in the West. As with the discussion of regional influences on central density, no proposition is made here to account for the regional variations in the density gradient. It remains for further research to inquire about the relative importance of potentially explanatory factors, such as variations in transportation systems, average income and land rents in the metropolitan areas.

Temporal effects indicate the pervasive decline of the density gradient during 1941 to 1976. There were particularly large decreases in the density gradient within the decade following World War II, when there was rapid expansion of the suburban housing stock. By the late 1950s, the pace of population suburbanization seems to have slackened, although the density gradient continued to decline. It would appear from these results that population deconcentration continued in Canadian metropolitan areas in the 1970s, although at a somewhat reduced pace. Stated in another way, there is no evidence from these data of faster central city growth than in the suburban areas or a reversal of the long-standing trend of deconcentration.

TABLE 8. MULTIPLE CLASSIFICATION ANALYSIS OF DENSITY GRADIENTS, 1941-1976

Variable	Number	Category Mean	Deviations Unadjusted	Deviations Adjusted
Metropolitan Population Size				
-250,000	54	.78	.12	.12
250,000-500,000	27	.67	.01	.01
500,000+	30	.44	-.22	-.23
City Age (in years)				
-10	20	.69	.03	-.15
11-30	29	.75	.09	-.06
31-50	24	.70	.04	.01
51+	40	.56	-.10	.11
Region				
Maritimes	20	.97	.31	.25
Quebec	16	.68	.02	.05
Ontario	51	.60	-.06	-.06
West	24	.51	-.15	-.11
Year				
1941	12	1.03	.37	.35
1951	15	.93	.27	.27
1956	14	.69	.03	.01
1961	16	.67	.01	.00
1966	16	.63	-.03	-.06
1971	19	.45	-.21	-.18
1976	19	.42	-.24	-.22
Sample Size	111			
Grand Mean		.66		
Multiple R				.756

Conclusion

We began this paper by arguing that population deconcentration was likely to have slackened during the 1971-76 period compared to the three previous decades. An initial examination of cross-sectional data on the mean density gradient and the mean central density suggested only minor diminuation in deconcentration. This first view, however, ignored the temporal process which includes the addition of new urban areas to the original set of cities at the same time each urban area became older. The thrust of this methodological observation leads us to examine multivariate analysis (MCA) with an effect for each year of observation.

Our primary conclusion from the MCA results is that there was a slackening in the decline of central densities and the density gradients within recent years, but without a reversal in long-term trends. In particular, the time period effect for 1976 shows continuation of the trend of lower central city densities and more dispersed population settlement. In summary, the pace of population deconcentration in Canadian metropolitan areas appears to have slackened within the last 10 or 15 years with less dramatic reductions in central city density and in the continuing dispersal of urban population. On the other hand, contrary to the expectations of some earlier forecasts (Bourne and Harper, 1974), these data do not show increased relative population growth in central cities. Some growth of households has occurred, primarily because of a reduction of the average number of people per household.

Placing this study in the context of other investigations of the metropolitan deconcentration of North American cities (Edmonston and Davies, 1976; Edmonston and Guterbock, forthcoming; Hawley, 1956; Mills, 1972b:101; Yeates and Garner, 1971:262-285), the dispersion of urban residential populations has occurred since at least late in the nineteenth century. This study of Canadian metropolitan areas shows that population dispersion, charted by a combination of central density and the density gradient, continued during the past three to four decades, although there appears to be some recent slackening in the rate of change. The exact explanation for the slackening rate of reduction remains to be explored. The introduction of this paper presented several temporal changes that would predict a diminution in deconcentration, although comprehensive examination of additional factors is needed in order to make a full accounting of the relative influence of possible variables.

Studies of urban population distribution in Canadian cities thus face interesting research challenges since it now appears metropolitan deconcentration has slackened. While such a major shift in population distribution warrants further attention to the major factors bringing about change, it is important to emphasize that additional empirical studies are needed to replicate the findings supported here. Additional work needs to focus on changes before 1941, and on a more complete accounting of why cities vary in their levels of central density and density gradient at a given time, why they vary their rates of deconcentration during a given period and why the deconcentration process varies in pace over successive time periods.

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