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SEX ROLES AND FERTILITY: WHICH INFLUENCES WHICH?

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Résumé — Cet article rend compte d'une étude pilote concernant les attitudes à l'égard des rôles des sexes et de la fécondité chez des adolescentes et des mères d'âge mûr de Toronto. Utilisant l'analyse de classification multiple, on à évalué, tout en éliminant les effets de la situation socio-économique et de la religion, l'importance relative de l'attitude à l'égard des rôles des sexes dans la prédiction des préférences quant à la dimension de la famille. Deux variables concernant l'attitude à l'égard des rôles des sexes — la valeur des enfants et l'activité extérieure — se révèlent importantes pour prédire les préférences en matière de fécondité tant des mères que des adolescentes de l'échantillon. De plus amples analyses conduisent à l'interprétation suivante: le système de motivations qui favorise les rôles maternels et domestiques au détriment de l'activité extérieure rémunérée se crée relativement tôt dans la vie et influence la formation de la famille après le mariage. L'article discute de la possibilité d'inférer de ces résultats des liens de causalité entre emploi et fécondité.

Abstract—This paper reports on a pilot study concerning sex role and fertility attitudes of teenage girls and middle age mothers living in Toronto. Using Multiple Classification Analysis an assessment is made of the relative importance of a number of sex role attitudes measures in predicting family size preferences, while controlling for the effects of socio-economic status and religion. Two sex role attitude variables — value of children and favourability to work — are important predictors of fertility preferences for both mothers and teenagers in this sample. Further analysis leads to the interpretation that the motivational system which favours the childbearing/home role and plays down employment roles is formed relatively early in the life cycle and continues to exert some influence on childbearing from marriage onwards. The implications of these results for making causal inferences about employment/fertility relationships are discussed.

Key Words—**sex roles, fertility, female employment**

The purpose of this paper is to review selected theoretical and empirical relationships between sex roles, family size preferences and fertility. The first part of the paper discusses two research issues: (1) Sex roles have been conceptualized in a variety of ways and, since most studies include only one measure, it is difficult to evaluate the relative importance of each measure for predicting family size preferences or fertility. (2) Existing studies are generally based on the assumption that sex roles affect fertility preferences and, hence, fertility. The role that fertility may play in influencing sex roles and family size preferences is still not clear.

The second part of the paper presents some empirical data from a Canadian pilot study designed to approach the two research problems indicated above. Specifically, the study attempts to assess the relative importance of a number of sex role measures in predicting family size preferences while controlling for the effects of socio-economic status and religion. In addition, the study examines the possibility that sex roles and family size preferences may be influenced by the number of children in the family as much as, or more than, they influence fertility.

It should be noted that fertility patterns in Canada are distinctive from those in the United

their husbands should be expected to take on an equal share in housekeeping and childcare, others may not; some may feel that if adequate childcare facilities are not available to them they can be founded through personal or community action, while others may react more passively. Finally, some may feel that they are able to handle both home and employment roles even with a large family, while others see themselves as incapable of doing both activities adequately.

Such definitions of whether extra-familial activities and large families are conflicting or not, rest, in part, upon what the woman perceives to be appropriate behaviour for women in general, and for herself in particular. Thus, her attitudes toward combining roles are crucial in her perception of a conflict and in her resolution of the conflict if she believes there is one (Hass, 1971). The implication here is that fertility researchers must give attention to sex role attitudes as well as to role participation and objective circumstances. The conceptualization and measurement of sex role attitude variables that have been used in fertility studies are discussed below.

Dimensions of sex roles related to fertility

Sex role attitudes have been conceptualized in at least the following three ways: (1) as *standards* — expectations regarding how the sexes ought to behave; (2) as *self-concepts* — self-evaluations of the degree to which one regards oneself as masculine or feminine in relation to the stereotype for one's sex, and (3) as *role motives* — interpretations of why one participates in a role, usually based on an evaluation of the costs and benefits (values) associated with the role activity.

Standards. A number of studies have included measures of sex role standards. A common format is to ask the respondent to rate her agreement with a series of statements which refer to sex-typed behaviour in a variety of situations. For example, "Husbands should always punish children" (parenthood role), "Wives should remain silent rather than argue with their husbands if they disagree" (marital role), "A woman should give up her job if it interferes with her family life" (labour force participation role). Such discrete items are typically scaled on a "traditional" to "modern" dimension. Standards reflecting a segregated role (husband and wife have separate, nonoverlapping duties), husband dominance (in financial decisions especially), and devotion to husband and children (few extra-familial involvements) are generally found to be intercorrelated and form the "traditional" attitude pattern. Moderate relationships between such attitudes and fertility have been found by Rosen and Simmons (1971); Hass (1971) and Scanzoni (1972).

Self-concept. Broverman and her colleagues (1975) have proposed that many of the traits generally attributed to women such as passivity and emotionality imply home-oriented, pronatalist behaviours, whereas the stereotype for men (aggressive, logical) emphasizes traits which are appropriate for instrumental, extra-familial activities. The extent to which women have incorporated the stereotypically female traits or stereotypically male traits into their personality should then have implications for their participation in familial and nonfamilial roles. This hypothesis has been investigated among Catholic women, among women students in a Catholic university, and for a larger cross-sectional sample of women in three social classes (Broverman et al., 1975). Selected relationships have been found between scores on a male cluster of traits ("competency" cluster) or on a female cluster of traits (a "warmth" cluster) and fertility or family size preferences. For example, among older Catholic women those with higher self-concept scores on the "competency" cluster have in general smaller families, even when education and work experience are controlled (Clarkson et al., 1970). And, women college students with higher "competency" scores desire fewer children (Broverman et al., 1972). Finally, in the larger cross-sectional study, (Broverman et al., 1975) found that both the "warmth" and "competency" self-concept clusters were related to reproductive behaviour in

may also have some bearing on the number of children born, but this influence may be negligible in the face of other more powerful determinants.

In contrast, actual family size may have a strong effect on sex role attitudes and behaviour. A woman with no children, or with only one or two, may be more inclined to become involved in extra-familial roles both because she has fewer time constraints and perhaps because there is not enough fulfilment in her home role. She may also be less dependent on her husband, since she has the time and energy to pursue her own interests. Finally, a woman with few children may seek to prepare herself for a less "traditional" role in anticipation of the time when her children have left home.

2) A second interpretation is that family size preferences, fertility and sex role attitudes may all be dependent upon certain background experiences. If this were the case, correlations between these variables would be "spurious" in the sense that they would disappear if the background variables in question were controlled. For example, female employment outside the home may be in response to economic pressures which also tend to reduce fertility. These same economic constraints might tend to lower family size preferences and to increase interest in satisfactions to be found in extra-familial roles.

If we wish to use survey data more effectively to understand whether sex role attitudes and family size preferences are in fact interdependent and serve to influence fertility, special precaution must be taken in both study design and in analysis. In the research presented in the next section of this paper we have attempted to bypass some of the problems inherent in using cross-sectional survey data to assess causal relationships. To explore the extent to which fertility may influence sex role attitudes and family size preferences, we have carried out two kinds of analysis: (1) The relationship between family size preferences and sex role attitudes is analyzed among young women who have not yet married, begun childbearing or entered adult roles of any kind. If sex role attitudes and family size preferences are intercorrelated among such girls we may better conclude that these two dimensions do form an integrated motivational complex which may have subsequent influence on actual family size. Our analysis in this regard is similar to that previously presented by Broverman et al. (1972), except that our analysis goes beyond the former by controlling a number of other relevant variables (i.e. religion, income level and the age of the girl) which may influence family size preferences. (2) The relationship between sex role attitudes and family size preferences is also analyzed among older women who have completed their childbearing, controlling for the number of children they have had. If the relationship continues to hold even taking into account the number of children in the family, then we have further evidence that family size preferences and sex role attitudes form a motivational complex which is independent of opportunities created by actually having raised many or few children.

PART II: THE TORONTO STUDY

The Data

The data described in this paper are based on a subsample from a larger interview study of mothers and their teenage sons and daughters carried out in metropolitan Toronto in 1972 (Pearl, 1974; Simmons and Turner, 1976). The interviews with mothers and their children covered a wide range of topics including sex role behaviour in the household, expectations and aspirations for the child's future roles, sex role stereotypes, self-concepts and role standards. The data base for the study was a criterion sample, selected according to family income (low or middle) and religious-ethnic origin (Protestant-British, Catholic-Italian or Jewish-European). For the purposes of this paper we have chosen to include only mothers and daughters in the analysis.[1] The resulting sample structure is shown in Table 1.

TABLE 2 SELECTED SAMPLE CHARACTERISTICS

	MOTHERS				
	Religion			Income Level	
	Protestant	Catholic	Jewish	Low	Middle
	(37)	(32)	(26)	(43)	(52)
Mean age	42.2	40.9	44.2	42.6	42.0
		(F= 3.45, p < .05)		(n.s.)	
Education					
% less than high school	16	69	19	37	33
% some high school or more	84	31	81	63	67
Mean % married life worked	39	42	20	40	31
		(F= 3.70, p <.05)		(n.s.)	
Mean number of children	3.00	3.64	2.77	3.12	3.17
		(F= 4.03, p < .025)		(n.s.)	
	DAUGHTERS				
	(32)	(23)	(15)	(30)	(40)
Mean age	15.7	15.5	16.0	15.7	15.7
		(n.s.)		(n.s.)	
Mean years schooling	10.0	9.7	10.5	10.0	10.0
		(n.s.)		(n.s.)	

A final sex role measure, "favourability to work," was included in an attempt to capture the attitude of each woman toward participation for herself or her daughter in extra-familial activities under certain restricting conditions.

We employed two measures of family size preferences in the interviews: one is the standard "ideal size" question ("How many children would you like to have?"), the other is a more detailed measure which assesses the range of different family sizes the respondent would be satisfied with. Problems in interpreting responses to the standard survey question regarding "ideal" family size have been discussed at length elsewhere (see George, 1973; Ware, 1973; Coombs et al., 1975). We would argue that the second and more complex measure we have developed may be more useful for the kind of exploratory research undertaken in this study. First, it avoids the problem that many individuals have in giving a single number as an ideal family size. Secondly, it allows rather precise measurement of the whole range of sizes that the respondent might consider "acceptable," fixing both upper and lower limits to that range. We suspect that the upper limit or "highest number of children satisfied with" should be an indicator of the family size beyond which the respondent would not continue to bear children. We carried out identical analyses with "ideal" family size and "highest number satisfied with." A similar pattern of results was obtained with both dependent variables, hence we have presented only the results for the "highest number satisfied with" variable in this paper.

Notes to Table 3

[a] The self-concept scale score is the average of the responses to 7 questions regarding personality traits which were rated by the respondents as significantly female stereotyped characteristics. In a previous study these items were also significantly rated as "socially desirable" personal characteristics (see Pearl, 1972). All 7 questions were asked using the same format which is as follows. "In comparison to others of my own sex, I am (MUCH LESS) (SOMEWHAT LESS) (EQUALLY) (SOMEWHAT MORE) (MUCH MORE) ....." The traits which ended this statement were: talkative, emotional, affectionate, sensitive, home-oriented, gentle, polite. Each response was scored as follows: much less (1), somewhat less (2), equally (3), somewhat more (4), much more (5). Analysis of intercorrelations between these items revealed that for mothers 95% of the correlations were significant at at least .05 level, and, for daughters 67% of the correlations were significant at at least the .05 level.

[b] The role standard score is the average of the responses to 8 attitude items regarding sex role proscriptions. Respondents were asked to rate their agreement with each item on a 40 point scale from 10 (low agreement) to 50 (high agreement). A high score indicates "traditional" sex role standards, a low score indicates "modern" sex role standards. The items were as follows:

(a) It is all right for a wife to express her opinions, but the husband should always have the final say.

(b) The father should have the primary responsibility for punishing the children.

(c) Marriage and family life should provide all the fulfillment that a woman needs.

(d) A wife should give up her job whenever it inconveniences her husband and children.

(e) A woman owes it to her husband to be a virgin when she gets married.

(f) It is better for a woman to keep silent than to argue with her husband when she disagrees with him.

(g) A married woman's most important task in life should be taking care of her husband and children.

(h) Girls should have to come home earlier at night than boys of the same age.

Analysis of inter-item correlations revealed that for mothers 86% of the correlations were significant at at least .01 level, and for daughters, 93% of the correlations were significant at at least the .01 level.

[c] The value of children score is the average of responses to 8 items regarding beliefs about having children. Respondents were asked to rate their agreement with each item on a 40 point scale from 10 (low agreement) to 50 (high agreement). The items were as follows: The bracketed form is that used for the sample of daughters.

(a) I feel that my child is (will be) an extension of myself.

(b) I think people are destined to reproduce -- it's part of the plan and purpose of life.

(c) For me having children is (will be) a necessary part of my self-fulfillment.

(d) Having a child is (will be) the most creative thing I have done (will do) in my life.

(e) Having a child brought (will bring) my husband and I closer together.

(f) My self-image as a real woman is (will be) confirmed by having a child.

(g) I feel that having a child turns girls into women and boys into men.

(h) I would (will) feel out of place if I didn't (don't) have children.

Analysis of intercorrelations between items revealed that for mothers 100% of the correlations were significant at the .01 level, and, for daughters, 78% were significant at the .01 level.

[d] Mothers were asked: "How favorable would you be about your daughter taking a paying job when she has preschool age children?"
Daughters were asked: "If it were not financially necessary, how would you feel about working at a part-time job when you had preschool age children?"
Respondents were asked to indicate their response on a 40 point scale from 10 (low favorability) to 50 (high favorability).

[e] Nine distinct questions were asked: "How would you feel if, in total, you actually raised ... no children at all? ... one child? ... two children? ... etc." In each case the respondent indicated her satisfaction on a 40 point scale, running from 10 (strong dissatisfaction) through 30 (neither satisfied nor dissatisfied) to 50 (strong satisfaction). The "highest number satisfied with" is the highest family size for which a score of 30 or higher was given.

Given that there is some overlap (intercorrelation) between variables, we will now turn to an analysis in which the power of each variable in "explaining" family size may be considered with the effects of the other predictor variables held constant. In addition we will include in the analysis two social background variables — income level and religious affiliation.

TABLE 5 RELATIVE IMPORTANCE OF BACKGROUND, AGE AND ATTITUDINAL FACTORS ASSOCIATED WITH FAMILY SIZE PREFERENCES FOR DAUGHTERS

Dependent variable: Highest number of children satisfied with

Predictor Variables	Beta Coefficients (1)	(2)	(3)	(4)	(5)	(6)	(7)
Background:							
Religion	.22	†	.21	.22	.28	.18	.21
Income level	.11	†	.14	.09	.12	.17	.19
Age:	.08	†	.08	.06	.08	.02	.01
Attitudinal:							
Self-concept	†	.04	.07	†	†	†	.07
Sex role standards	†	.13	†	.19	†	†	.10
Value of children	†	.14	†	†	.29	†	.21
Favorability to work	†	.28	†	†	†	.32	.27
R^2	.08	.15	.08	.11	.16	.18	.24

† Omitted from this run.

In Tables 4 and 5 the beta coefficients derived from the MCA programme are tabulated for various sets of predictor variables. These betas are indicative of the relative importance of each predictor in accounting for variance in family size preferences of mothers (Table 4) and of daughters (Table 5). Again, we have used "highest number of children satisfied with" as the measure of family size preferences. Seven separate regressions are shown in each table (see columns 1 to 7) using different combinations of predictor variables. The interpretation of these various regressions will be discussed in a subsequent section. For the moment we may focus attention on column 7 in each table which shows the relative power of each predictor in an equation including background, behavioural and attitudinal measures.

When all predictor variables are included in a single regression the overall prediction of family size preferences is modest (R^2 = .24) for both mothers and daughters. However, the generations differ considerably in the ordering of importance of the sex role attitude variables in explaining family size preferences. For mothers, value of children score, sex role standards and favourability toward work are almost equally important contributors to prediction of family size preferences (betas .21, .18 and .15). For daughters, attitudes toward work seem to be relatively more important than value of children attitudes and both are clearly more important than sex role standards. For both generations sex role self-concept is comparatively the weakest predictor among the attitude variables.

In MCA analysis the beta coefficients do not indicate the range of variation of family size preferences between those scoring "high" or "low" on any attitude measure. Neither do they show the direction of the relationship between each predictor and the dependent variable. Tables 6 and 7 present the unadjusted and adjusted means on family size preferences for those respondents scoring "low" (below the median) and "high" (above the median) on each sex role attitude measure.

As expected, the unadjusted means presented in these two tables simply correspond to the correlations shown in Table 3. The key issue is whether the *adjusted* means are different and, if so, how. For three of the sex role attitude measures the adjusted means are only minimally different from the unadjusted mean family size preferences for women scoring "high" or "low"

on each variable. These variables are self-concept (which remains a poor predictor), value of children and favourability to work (which remain relatively "good" predictors). This is true for both mothers and daughters.

One unexpected finding occurred for mothers. Sex role standards initially had no relationship with family size preferences (zero-order correlation between the two variables is .01; see Table 3). After adjustment for intercorrelations with other predictor variables, however, role standards became related to family size preferences *but in a direction opposite to that expected.* That is, once the other predictor variables are controlled in the regression procedure, "modern" sex role standards become associated with *higher* family size preferences (see the adjusted means in Table 6). Further analysis led to a partial explanation of this unusual finding. We found that the unexpected negative correlation between sex role standards and family size preferences emerged most clearly when the value of children variable was controlled. For example, among mothers who place a relatively high value on the rewards of childbearing and rearing, there are two subgroups; a larger group of mothers who (consistent with their value of children scores) favour a relatively traditional role for women, and a smaller group of mothers who (inconsistent with their value of children scores) favour a relatively modern role for women. Not surprisingly, the larger group of mothers with high value of children scores *and* with traditional role standards favour relatively large families. Surprisingly, however, the smaller group of mothers who combine high value of children scores with modern role standards want even more children. These women seem to be saying that, for them, greater decision making power and a less segregated role for the woman is fully consistent with having more children. In some sense this pattern is reminiscent of the "superwoman" syndrome we have noted with regard to a minority of the daughters in the same sample. This syndrome refers to daughters who have very high career aspirations and who also want relatively large families. Perhaps these two can be best combined in the context of marriage in which the wife has important decision making power, in which the husband is willing to share in childcare, and so on. These hypotheses cannot be explored further with the data at hand, hence they remain interesting topics for future investigation.

Causal relationships between sex role variables and family size attitudes. As Blalock (1961) has noted, inferences regarding causal hypotheses are possible using cross-sectional data such as that available in the present study. The following analysis is guided by two questions: (1) Do any of the sex role attitude variables add to the proportion of variance accounted for in family size desires when the effects of social background and behavioural factors are controlled? If the answer is yes, we can infer that those sex role attitude variables are, at least in part, independent predictors of family size desires. (2) Are sex role attitudes related to family size desires when number of children born is controlled? If so, we may infer that sex role attitudes and family size desires form part of a motivational system which may act upon fertility and which is not merely a consequence of number of children already born.

Regarding the first issue our results indicate that:

(a) The background variables — religion, income level and (for mothers) per cent of married life worked, number of children, and (for daughters) age — taken together do provide a low prediction of family size desires (R^2 = .14 for mothers and .08 for daughters). See column 1 in Tables 4 and 5.

(b) The overall prediction provided by the attitude variables in themselves is approximately the same as that for background factors (R^2 = .14 to .15). See column 2 in Tables 4 and 5.

(c) Overall prediction increases slightly when the sex role attitude variables are added one at a time to the background variables. See column 3 to 6. However, the greatest overall prediction is obtained when both attitude and background variables are considered together (R^2 = .24 for both mothers and daughters). See column 7 in each table.

found with respect to it. Conceptually, the role standards refer in part at least to a commitment to one's family in terms of wife and mother obligations. One would expect that they would show an "intermediate" association with family size preferences, since the importance of such obligations would likely be increased in a large family. This prediction was generally borne out in the analysis of the sample of daughters. However, in the case of the mothers it was found that sex role standards in themselves were not related to family size preferences. Moreover, when other variables, such as "value of children" were controlled, the sex role standards among the mothers became inversely associated with family size preferences. Thus, for example, within the group of women with relatively high value of children scores, the minority who were relatively "modern" on sex role standards wanted the most children. Is this finding an anomaly of the present small pilot sample, or do some women really like to see themselves as "superwomen" who can have extra-familial and career commitments, a relatively high status position in family decision making, and more children than other women as well? Only further investigation can answer this question. However, some findings in existing research may support indirectly the "superwoman" hypothesis. For example, Broverman et al., (1975: 15) found that the women in their sample who were high on both the "warmth" self-concept subcluster (female stereotyped traits) and high on the "competency" self-concept subcluster (male stereotyped traits) had the most children of all.

Several features of the present study should be considered in evaluating the results. Firstly, this is an exploratory investigation involving a relatively small number of cases. Confidence in the results from MCA analyses is greatly diminished with small samples. As well, some researchers have reservations about how the betas from the MCA programme should be interpreted, since the regressions in the MCA procedure are for each category of the variable and not for the variable as a whole. Secondly, some of the measures used in the study are still in the early stages of development. For example, the value of children scale taps only a limited number of motives for parenthood. It might be possible, therefore, that a respondent would answer negatively to all the items and still place a high value on children for other reasons. Recent research by Fawcett and his colleagues (1974) indicates that there is a wide range of reasons for desiring to have children. Finally, self-rating scales such as the self-concept measure used in this study can be problematical when interindividual comparisons are made. Perhaps the study has not measured accurately the sex role variables whose association to fertility it wished to ascertain.

Clearly, further research is indicated using larger samples and designs which permit more direct examination of the extent to which the motivational system of sex role attitudes and family size preferences actually influences fertility. We are limited here to making causal inferences based on a careful analysis of cross-sectional data. As evidenced in the discussion above, the findings in this paper suggest that some sex role variables and family size preferences do form a motivational system. Moreover, indirect evidence in our analysis also argues that this motivational system emerges independently of actual role experiences in marriage brought about by the opportunities and constraints created by having actually borne more or fewer children. We found that the motivational system is intact among young women even before they have any children or enter marriage and adult work and home roles. We also found that the motivation system was evident among older married women who had finished their childbearing, even when the number of children they had raised and other relevant background experiences (such as their actual work experience) were controlled. Thus, insofar as there is a relationship at all between the attitudes in this "motivational complex" and actual fertility, it would appear that it is the attitudes which influence fertility and not vice versa. Since this is what most investigations on this topic have assumed, researchers pursuing this hypothesis should be encouraged. At the same time one should not be sanguine. Among the mothers in our sample, the relationship between family size preferences and actual completed

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RESEARCH NOTES

OPTIMUM VALUES OF LIFE TABLE FUNCTIONS

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Résumé — On sait que la fonction l_x a une forte pente négative à l'âge zéro et cette pente continue à diminuer jusqu'à ce qu'elle devienne très faible à un certain âge et commence à augmenter par la suite. De même, l'intensité de moralité μ_x prend une valeur minimale et l'espérance de vie $\mathring{e}_x$ atteint une valeur maximale aux âges appropriés. Ces trois catégories d'âge forme une suite avec l'âge correspondant à l'espérance de vie maximale comme la moindre. Les estimations de ces valeurs à partir de plusieurs tables de mortalité indiquent des relations inverses avec l'espérance de vie à la naissance.

Abstract—It is known that the function l_x has a steep negative slope at age zero and the steepness continues to diminish till it becomes smallest at some age and begins to increase thereafter. Similarly, the force of mortality μ_x assumes a minimum value and the life expectancy $\mathring{e}_x$ attains a maximum value at appropriate ages. These three ages form a sequence with the age corresponding to the maximum life expectancy as the smallest. Estimates of these values from several life tables demonstrate inverse relationships with expectation of life at birth.

Key Words—**maximum, minimum, mortality measures**

The Problem

The characteristics of life table functions are quite well known but investigations on some of the crucial values of these functions and their interrelationships have not yet been carried out. The importance of such an inquiry need not be over-emphasized in view of the fact that any exercise on graduation of life table functions must take into account the order and spacing of these parametric values.

The object of this investigation is to isolate three important characteristics of a life table, namely:

1) the age at which the derivative of l_x, i.e., of the proportion surviving from age zero to age x (which is uniformly negative) assumes its maximum or negatively minimum value;
2) the age at which μ_x or the force of mortality $-(dl_x/dx)/l_x$ is minimum, and;
3) the age at which the expectation of life $\mathring{e}_x$ is maximum.

In other words, the paper deals with the derivation of the minimum value of μ_x and the maximum values of each of the two other functions, namely, $\mathring{e}_x$ and dl_x/dx. Since these functions are known to be continuous and differentiable, the desired values can be obtained by equating the first derivatives of the respective functions to zero. The solutions thus obtained are known as local extremums in the mathematical literature, and whether a given extremum is a maximum or a minimum is determined by the sign of the second derivative of the function at that point. In the particular case of a function having only one extremum (maximum or minimum), the local extremum is also equivalent to the only extreme value of the function. Needless to point out that the existence of extreme values is not a necessary characteristic of a function; however, each of the functions chosen for this study has one extremum and as will be shown later, these optimum values exhibit certain interesting features.

is zero at $x = \tilde{x}$, so that

$$l_{\tilde{x}}\mu_{\tilde{x}}^2 = \left[\frac{d^2\, l_x}{dx^2}\right]_{x=\tilde{x}} \tag{8}$$

Now, the derivative of l_x is uniformly negative, and because of the large force of mortality at age zero, assumes a maximum value at, say $x = \bar{x}$. Correspondingly, the second derivative of l_x is positive at age zero, decreases thereafter till it becomes zero at $x = \bar{x}$.

Since the left and therefore the right hand side of (8) are positive, it is clear that the minimum value of the force of mortality is reached before the derivative of l_x assumes its maximum or its smallest negative value. In other words,

$$\tilde{x} < \bar{x} \tag{9}$$

and combining (9) with (5), the following inequality relationship,

$$\hat{x} < \tilde{x} < \bar{x} \tag{10}$$

is established.

Estimation of $\bar{x}$, $\tilde{x}$ and $\hat{x}$

The exact ages at which the three optimum conditions are met cannot be obtained since none of the life table functions l_x, μ_x or $\mathring{e}_x$ can be expressed in terms of easily differentiable mathematical functions. Approximate solutions were however, obtained earlier (Mitra, 1974) for ages corresponding to maximum life expectancies for different life tables. The method used was to find the point of intersection of two freehand curves obtained by plotting the values of $\mathring{e}_x$ and $1/\mu_x$ at early childhood ages. The values of μ_x can be determined from probabilities of dying ${}_nq_x$ in the age interval x to $x + n$, because of the well known relationship

$$1 - {}_nq_x = e^{-\int_0^n \mu_{x+t}\, dt} \tag{11}$$

The exponent $\int_0^n \mu_{x+t}\, dt$ can be approximated by $n\mu_{x+\frac{n}{2}} (n < 5)$

for the age interval 1 to 20 where the force of mortality is relatively small and finally, disregarding squares and higher powers of μ_x, (11) can be reduced to

$${}_nq_x = n\mu_{x+\frac{n}{2}} \tag{12}$$

Estimated values of the reciprocal of the force of mortality obtained from known values of age specific mortality rates can be plotted together with the values of the expectation of life. The x co-ordinate of the point of intersection of the curves of the two functions provides an estimate of $\bar{x}$ or the age at which the life expectancy assumes its maximum value. This method, rather than that based on numerical analysis is chosen for reasons of operational simplicity.

Next, because of (12), the age, says x', at which ${}_nq_x$ is minimum, is approximately related to $\tilde{x}$ by the following equation,

$$\tilde{x} = x' + n/2 \tag{13}$$

The method of finite differences can then be used to determine x' for which ${}_nq_x$ values for $n = 5$ and $x = 5$, 10 and 15 are sufficient, since x' can generally be located around age ten. Using the conventional notation Δ for successive differences, so that

$$\Delta\,({}_nq_x) = {}_nq_{x+n} - {}_nq_x$$

and

$$\Delta^2\,({}_nq_x) = \Delta\,({}_nq_{x+n}) - \Delta\,({}_nq_x)$$

etc., the solution for $\tilde{x}$ can be found by equating the first derivative of the polynomial approximation of ${}_5q_x$ to zero, solving for x' and adding 5/2 to the result. (According to

tables are systematic and small. Like $\hat{x}$, $\tilde{x}$ and $\bar{x}$ are also inversely related with life expectancy, and cover a range of 13.2 to 11.1 and 13.7 to 11.2 respectively over the range of $\mathring{e}_0$ used in this analysis. As noted in (9) $\tilde{x} < \bar{x}$, but the difference is rather small in most cases.

Hopefully, future researchers will be studying the variations of these optimum values among real life tables corresponding to different countries at different times as well as among other model tables prepared by Coale and Demeny (1966). It is expected that the distribution of these important parameters will provide interesting guidelines for purposes of graduation of life table functions or for the derivation of estimates of these functions from mortality statistics subject to errors resulting from various sources.

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