

If Mowat & Davis Are Correct, Then Teaching is Hard

A Response to Elizabeth Mowat & Brent Davis

KRIS GREEN & BERNARD RICCA

Saint John Fisher College (USA)

Mowat & Davis (this issue) present a model of learning mathematics that relies heavily on ideas from network (or graph) theory. The important questions (to us, at least) concern the dynamics of the nodes and links. Answers – even tentative ones such as we present here – to these questions lead to a second set of questions concerning the implications of these answers to teachers and researchers.

We present here some tentative ideas about the dynamics of links and nodes within the proposal of Mowat & Davis. Some of the discussion that led to these ideas occurred on a wiki (Green & Ricca, n.d.); we omit some of the details here for space reasons and refer the interested reader to the wiki. We are making no claim that the dynamics presented here are optimal or complete. Instead, we present these claims only to explore some implications for teaching and research. Further, while there are several possible ways to assign meaning to nodes and links, we find that the dynamics of the nodes and the links to be of more interest, and so we will make only minimal definitions of nodes and links.

Nodes and Links in Maths Learning

We take nodes to be things such as algorithms, experiences or concepts - those chunks that can be applied to a problem or a situation. With this definition in mind, we consider the following operations on nodes:

- Create a new node from experience
- Redefine the internal structure of a node
- Create a new node from a collection of existing nodes

We do not consider the destruction of nodes; instead we state only that it is possible that all of a node's links could be removed, leaving the node to be apparently forgotten. However, we still have the open question as to whether there is value in deleting a node, or if it is even possible to do so. In addition, we consider the following operations on links:

- Creation of a link
- Breaking and removing a link
- Replacing one end of a link with a different node
- Changing the nature of the link

In all of these cases, there is the implication that nodes and links have a sort of meta-stability to them: Links and nodes are neither necessarily permanent nor unchanging, but they can remain relatively unchanged over time-spans that are long compared to a single problem or even a single course. We think that this set of operations is sufficient to examine all of the processes of knowing, and we now explore the implications of learning as a network.

Dynamics of nodes

We begin by examining nodes. New nodes, in our view, come from experiences. These experiences need not be "firsts", as in the first bird we ever encounter, but need to be something that triggers the creation of a new category or process. (We also, at this point, are not concerned with the process of creation, although that is probably a very interesting question.) It is possible (Minsky, 1986) that the new category inherits several properties from an old category, as when a young child moves from knowing a particular dog to knowing that there exists a class of animals all of whom are dogs.

All concepts and understandings have some structure. Even if a concept turns out to be a collection of nodes, one can consider the collection to be a single node with internal structure. It is possible for this internal structure of a node to change. As an example of the change of internal structure of a node, consider the maths network of a beginning algebra student. This student certainly has some method (or collection of algorithms) with which to perform multiplication. Most likely, however, this method is one of the traditional algorithms and the student probably understands that algorithm as repeated addition. In our experience and the experience of others (Devlin, 2009; Jacobson, 2008),

when adults are asked to explain why $3 \times 3 = 9$, their explanations often involve repeated addition. The understanding of multiplication as repeated addition does not connect to problems such as “ x times x ” (which partly explains why the most common reduction of that expression by beginning algebra students is $2x$.) The node labeled “multiplication” must change in this case for the student to learn algebra.

Here, the thought of nodes having structure is an important one. Nodes with structures, we posit, are roughly equivalent to conceptual understandings, while nodes without structure are simply facts or algorithms, what Bruner (1964) called “iconic representations”. These two types of nodes are used much differently when applied to a problem.

New nodes can be created from a collection of existing nodes. For example, many algorithms can be used to calculate the result of the operation of multiplication. However, in addition to those algorithms, a new understanding (a.k.a., new node) of multiplication can be created; this new understanding is a concept which contextualizes all the algorithms.¹ In this case, the new conceptual node can look both ways: at the problem to be solved and at all of the possible tools. Collections of nodes, without the conceptual structure, can be used to attack a problem, but the process seems to be one of successive attempts to link a (structureless) node with the problem to be solved. On the other hand, a conceptual (structured) node looks both at the problem and at the algorithms in order to choose an algorithm that is most appropriate; the choice is motivated by what works and by other things such as interest, elegance, brevity, etc.

Dynamics of links

We consider links to be the various relationships between the nodes. While we recognize that there are many possible relationships between nodes, we are not interested in those details here.

It is an interesting and not trivial question why a link is created at all. We can make statements such as “humans are sense-making creatures” to hide this difficulty, but such statements yield no insight. Our conversations have led us to believe that links are created under three conditions: First, a learner must be motivated to create a connection between the two nodes. Second, the learner must possess a particular logic to establish the relationship. (The type of logic we mean is probably more like Piaget’s operatory logic than the formal logic of philosophy or mathematics.) Third, the resulting link must provide some additional benefit to the learner, for example by providing additional power or reducing cognitive load. The necessity of the first is well known to teachers; the second and third will get additional consideration below.

Examples of links seem to fall into three categories: Absolute, conditional or contextual links. Absolute links always exist, although conditions can override them (e.g., the teacher wants us to think a certain way, despite my personal beliefs). Conditional links are activated (used) only in certain circumstances, and their use is

¹ We examine this idea more in our wiki; See <http://mathlearningasnet.pbworks.com> and more details about this in the “open forum” in this volume (p. 106).

motivated by situations external to the learner (such as the desires of a teacher). Contextual links also are links that are activated only in certain circumstances, but the use of a contextual link is motivated by processes internal to the learner.

One example of conditional links comes from a recent paper by Ibrahim (2009), who finds that physical science students may “know” what is expected by a teacher but still fail to believe those ideas. In this case, and probably in other cases, students may give the teacher “what he wants” without changing their own beliefs.

The context for activating contextual links is an internal match to a problem. Consider the example above about reorganization of multiplication algorithms into a new multiplication node. In that example, the reorganization of algorithms into a new understanding of multiplication recontextualized the various algorithms by which multiplication can be performed: a particular algorithm will be chosen for a particular problem based on the features of the problem. This is different from the application of an algorithm chosen through a process of trial and error.

The creation of a link is a common enough experience. Often two nodes exist, such as the music student who is aware of eighth notes but has never linked those to the mathematical concept of eighths; the creation of that connection makes a substantial difference in the place of mathematics in the student’s life, as well as connecting one set of tools to another.

The removal of a link is problematic, and while we recognize that sometimes links must be removed, it is not clear why or how they are removed. For example, while most high school students know that it is incorrect to claim $(2+1)^{1/2} = 2^{1/2} + 1^{1/2}$, many of those students will still claim that $(x+1)^{1/2} = x^{1/2} + 1^{1/2}$, which indicates (possibly) that students think all functions of x are linear; this error persists often through calculus. It is also possible that students do not have a category of “nonlinear functions” to which they can connect the square roots, and hence are unwilling to break the link; other explanations are available.

Changing the nature of a link is implicated in creating a node from a collection of nodes, however, there is more to say. Whereas a metaphor (Lakoff and Johnson, 1980) is a one-directional link from source to target, the reorganization of a collection of multiplication algorithms into, for example, a single unified multiplication concept changes some unidirectional links to links that point in two directions.

Implication for Teachers and Teaching

While there are many possible implications for teaching, we focus here on one in particular: What a teacher must do to facilitate learning depends upon the student’s nodes and structures. Here, however, the “nodes and structures” include not only the knowledge directly pertaining to the problem, but also the operator structures that a student possesses. Hence, a first step in teaching maths is for a teacher to understand the existing network of a student’s maths abilities and their so-called metacognitive abilities. While this sort of formative assessment is talked about in many places, we note that there is a refinement of formative assessment that is possible using the network model:

The projection of the student network onto an appropriate expert model may be useful in helping a teacher; this seems to be at the heart of the learning progressions work. This projection requires that the teacher have one or more expert models (a.k.a., *content knowledge*) and the ability to project a student model onto that and use that information (a.k.a., a new take on *pedagogical content knowledge*). However, there is some additional understanding necessary: a teacher must understand how a student may construct and reorganize new nodes and links (a.k.a., a new take on metacognition).

It is quite likely, for example, that studies concerning discrepant events (see, for example, Tsai & Chang, 2005, for an example,) have data showing discrepant events do not always promote learning. In the network model, a discrepant event attempts to remove a link rather than changes nodes. Students who have no other node to connect with will find no way forward, and will (likely) reconnect the original link. For example, many people believe that summer occurs when the earth is closer to the sun than it is in winter. Teachers often attempt to break the link between “closer” and “summer” by pointing out that the southern and northern hemispheres have different seasons at a given time. However, even if this fact breaks a link, if there is no other explanation, then the broken link may reform. Unless an alternative explanation exists, and unless the link to that alternative can be established, it is likely that the students may restructure their knowledge by ignoring the discrepant event.

Because of this type of situation, teachers must consider what types of learning processes students possess. Given that there are different ways of reorganizing knowledge networks (i.e., different students have different operatory structures), it is important for a teacher to understand these different possible processes, and to assess the current student understanding *with respect to these processes*, before beginning instruction. Without this understanding, it is unlikely that instruction will be worthwhile. Our claim is that this use of the network model can inform how we traditionally do formative assessments and curriculum planning: Since we do not perform appropriate formative assessments, teaching and curriculum planning are really shots in the dark.

Further, changes in network structure come with a cost. For example, students often benefit from the highlighting of some intermediate result. In Calculus, for example, there is a rule for finding the derivative of a variable raised to some constant power. This rule is generally easier – and much shorter – than always returning to the definition of derivative and deriving the necessary result, but requires additional memorization beyond the original definition of a derivative. This perhaps yields insight into the power of mathematical proof and the nature of links. A proof indicates some intermediate result *always* is applicable, resulting in rather more permanent and broadly applicable links. These proven structures may be more easily accessible to students.

For example, the power rule for derivatives, by virtue of being provable (in the mathematical sense) is always applicable to power functions. This is unlike, for example, the intermediate result shared by many students that “multiplication always makes things bigger”. Note that, as seen in this example, not all links are equal: A proof takes a

collection of nodes and their supposed links and creates an entity more stable against potential future experiences.

Future Directions

The notion of maths learning as a network leads to many interesting and potentially important questions. In addition to the questions explored above and some explored in our wiki (Green & Ricca, n.d.), we have:

- Does a student need to be aware of their own operatory structures in the same way that they must be aware of their content knowledge? If so, what does this say about the current push for *learning progressions* in science teaching?²
- Some studies seem to indicate that there is a threshold of exposure for learning. In other words, it is perhaps necessary to encounter something a certain number of times before a link to that entity can be established. Is there such a threshold for learning, and if so, can it be appropriately measured?
- Why do students approach “missing information” problems in a different ways?³
- Does a student’s use of *marginal analysis* in problems serve as an indicator of the structure of nodes?

The idea of maths learning as a network has potential, then, to lead to additional insights into teaching and learning. We look forward to seeing further work in this area.

References

- Bruner, J. (1964). The course of cognitive growth. *American Psychologist*, 19, 1-16.
- Devlin, K. (2008, June). Devlin’s Angle: It ain’t no repeated addition. Retrieved 16 December 2009 from http://www.maa.org/devlin/devlin_06_08.html.
- Green, K. & Ricca, B. (n.d.). Response to Mowat & Davis. Retrieved 16 December 2009 from <http://mathslearningasnet.pbworks.com>.
- Ibrahim, B., Buffler, A, Lubben, F. (2009). Profiles of freshman physics students’ views on the nature of science. *Journal of Research in Science Teaching*, 46(3), 248-264.
- Jacobson, E (2009). In my opinion: Too little, too early. *Teaching Children Mathematics*, 16 (2), 68-71.
- Lakoff, G., & Johnson, M. (1980). *Metaphors we live by*. Chicago: University of Chicago Press.
- Minsky, M. (1986). *The society of mind*. Simon and Schuster: New York.
- Tsai, C-C., Chang, C-Y. (2005). Lasting effects of instruction guided by the conflict map: Experimental study of learning about the causes of the seasons. *Journal of Research in Science Teaching*, 42 (10), 1089 – 1111.

² See the special issue of *Journal of Research in Science Teaching* (volume 45, number 6) on learning progressions.

³ On the wiki, we briefly examine an example of this.

About the Authors

Barney Ricca is Director of the Graduate Program in Mathematics, Science and Technology Education at Saint John Fisher College. A physicist by training, his research involves complexity sciences with particular focus on teaching and learning. He teaches graduate courses in science and science education. He is currently the Chair of the Chaos and Complexity Special Interest Group of the American Educational Research Association. When not teaching, reading or writing, he can often be found bicycling or eating.

Kris Green is an associate professor in the Mathematical and Computing Sciences Department at Saint John Fisher College. His research focuses on technology and writing in mathematics and the sciences. He also teaches a course in world building as an integrated experience in the sciences for K-12 teachers. Outside of his office, he can often be found wearing a gi and waving weapons around at a local Isshinryu dojo or “studying” science fiction films and literature.

© Copyright 2010. The authors, KRIS GREEN AND BERNARD RICCA assign to the University of Alberta and other educational and non-profit institutions a non-exclusive license to use this document for personal use and in courses of instruction provided that the article is used in full and this copyright statement is reproduced. The authors also grant a non-exclusive license to the University of Alberta to publish this document in full on the World Wide Web, and for the document to be published on mirrors on the World Wide Web. Any other usage is prohibited without the express permission of the authors.